Math 425, Spring 2011 **Jerry L. Kazdan**

Problem Set 0 [Rust Remover]

DUE: Never.

1. Let $u(t)$ be the amount of a radioactive element at time t and say initially, $u(0) = A$. The rate of decay is proportional to the amount present, so
$$\frac{du}{dt} = cu(t),$$
where the constant c determines the decay rate. The *half-life* T is the amount of time for half of the element to decay, so $u(T) = \frac{1}{2}u(0)$. Find c in terms of T and obtain a formula for $u(t)$ in terms of T.

2. Let $\int_0^x f(t)\,dt = e^{\cos(3x+1)} + A$, where f is some continuous function. Find f and the constant A.

3. Say $w(t)$ satisfies the differential equation
$$aw''(t) + bw' + cw(t) = 0, \qquad (1)$$
where a and c, are positive constants and $b \geq 0$. Let $E(t) = \frac{1}{2}[aw'^2 + cw^2]$.
   a) Without solving the differential equation, show that $E'(t) \leq 0$.
   b) Use this to show that If you also know that $w(0) = 0$ and $w'(0) = 0$, then $w(t) = 0$ for all $t \geq 0$.
   c) [Uniqueness] Say the functions $u(t)$ and $v(t)$ both satisfy the same equation (1) and also $u(0) = v(0)$ and $u'(0) = v'(0)$. Show that $u(t) = v(t)$ for all $t \geq 0$.

4. Say $u(x,y)$ has the property that $\dfrac{\partial u}{\partial y} = 0$ for all points (x,y) and that $u(x,0) = \sin 3x$. Find $u(x,y)$.

   What if instead u satisfies $\dfrac{\partial u}{\partial y} = 2xy$?

5. A function $u(x,y)$ satisfies $u_x + 3u_y$=0. Find a change of variables
$$x = as + bt$$
$$y = cs + dt$$
so that in the new (s,t) variables u satisfies $\dfrac{\partial u}{\partial s} = 0$.

[Last revised: January 15, 2011]