

Math 425
Assignment 3

Dr. DeTurck
Due Tuesday, February 9, 2010

Reading: Textbook, Chapter 1, skim through Chapter 2.

1. Take a moment to review the divergence theorem from vector calculus, then work the following problem: Suppose $V(x, y, z)$ is a vector-valued function defined everywhere in 3-dimensional space. Further, suppose that V is differentiable and that

$$\|V(x, y, z)\| \leq \frac{1}{1 + (x^2 + y^2 + z^2)^{3/2}}$$

for all (x, y, z) . Show that

$$\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \nabla \cdot V(x, y, z) \, dx \, dy \, dz = 0.$$

(Hint: Apply the divergence theorem on a ball of large radius)

2. Solve the wave equation (for an infinite string) $u_{tt} = c^2 u_{xx}$ with initial conditions $u(x, 0) = \ln(1 + x^2)$ and $u_t(x, 0) = 4 + x$.
3. (The dulcimer) Solve the wave equation $u_{tt} = c^2 u_{xx}$ with initial conditions $u(x, 0) = 0$ and $u_t(x, 0) = g(x)$, where $g(x) = 1$ if $|x| < a$ and $g(x) = 0$ for $|x| \geq a$. This corresponds to hitting the string with a hammer of width $2a$. Draw sketches of snapshots of the string (i.e., plot u versus x) for $t = \frac{1}{2}a/c$, $t = a/c$, $t = \frac{3}{2}a/c$, $t = 2a/c$ and $t = \frac{5}{2}a/c$.
4. In the preceding problem, for each value of t , determine the maximum value of $u(x, t)$ (your pictures should help!).
5. Solve $u_{xx} + u_{xt} - 20u_{tt} = 0$, with initial conditions $u(x, 0) = \phi(x)$, $u_t(x, 0) = \psi(x)$. (“Factor” the operator the way we did for the wave equation.)
6. Show that, for any function $u(x, t)$ that satisfies the wave equation $u_{tt} = u_{xx}$ (with $c = 1$), we have

$$u(x + h, t + k) + u(x - h, t - k) = u(x + k, t + h) + u(x - k, t - h)$$

for all x, t, h, k .

7. Solve $u_{tt} = 9u_{xx}$ on $0 < x < \pi/2$, with $u(x, 0) = \cos x$, $u_t(x, 0) = 0$, $u_x(0, t) = 0$, $u(\pi/2, t) = 0$.
8. Textbook page 17, problems 3, 10; page 27, problems 2, 5.