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Example: A smooth function on $\mathbb{R}^2$ with exactly two critical points, both strict local minima.

It is not entirely obvious that such a function exists.
Begin with:

$$h(s,t) = (s^2-1)^2 + t^2$$

This has local min at $(\pm 1, 0)$ and a saddle at $(0,0)$. We want to remove the saddle from the picture.

Push the minima up a bit, and push the saddle down: $u = s$, $v = t + 3 - \frac{4}{1+s^2}$ so $s = u$, $t = v - 3 + \frac{4}{1+u^2}$.

$$g(u,v) := (u^2-1)^2 + \left(v - 3 + \frac{4}{1+u^2}\right).$$

Map the upper half-plane to the whole plane: $x = u$, $y = \ln v$, so $u = x$, $v = e^y$

$$f(x,y) = (x^2-1)^2 + \left(e^y - 3 + \frac{4}{1+x^2}\right).$$

A *polynomial* example – found by Jared Weinstein

$$f(x,y) = (x^2-1)^2 + (x^2y - x - 1)^2$$