Problem Set 9

DUE: Thurs. Nov. 15 in class. [Late papers accepted (without penalty) until 1:00 PM Friday.]

Chapters 9.6, 7.1-7.2 in Burden-Faires
As we have seen, a matrix A, usually not square, often arises as a matrix of data and we were led to consider he eigenvalues and eigenvectors of the symmetric matrix A^*A. This is treated in Section 9.6 of Burden-Faires. The *singular values* of A are the eigenvalues of A^*A. Some elementary basic facts are collected in Theorem 9.26.
Note If a matrix is not a data matrix, its columns (or rows) are *not* expected to be normalizes to have mean 0. In Section 9.6 Burden-Faires never normalize the columns (or rows).

Problems, most of them short.

1. Since the data might be presented as either the rows or columns of A, much of the information in A^*A and AA^* must be the same. Show that if $\sigma \neq 0$ is an eigenvale of A^*A, then it also an eigenvalue of AA^* – and find a formula for the corresponding eigenvector.

2. Burden-Faires Sec. 9.6 #2d

3. Burden-Faires Sec. 9.6 #5

4. Burden-Faires, Sec. 7.1 #4c)

5. Burden-Faires, Sec. 7.1 #6

6. Burden-Faires, Sec. 7.1 #7

7. Burden-Faires, Sec. 7.1 #9

8. Burden-Faires, Sec. 7.1 #10

9. Burden-Faires, Sec. 7.2 #2a.f)

10. Burden-Faires, Sec. 7.2 #6a,f)

11. Burden-Faires, Sec. 7.2 #8a,f)

12. Burden-Faires, Sec. 7.2 #12

13. If A is a square matrix with $A^{18} = 0$ (so A is nilpotent), show that

$$(I - A)(I + A + A^2 + \cdots + A^{17}) = I.$$

so $I - A$ is invertible and we have the interesting formula

$$(I - A)^{-1} = I + A + A^2 + A^3 + \cdots$$

14. Burden-Faires, Sec. 7.2 #14b)

15. Burden-Faires, Sec. 7.2 #15

16. Burden-Faires, Sec. 7.2 #16

[Last revised: November 10, 2018]