Math 320, Fall 2018 **Jerry L. Kazdan**

Problem Set 6

Due: Thurs. Oct. 25 in class. [Late papers accepted (without penalty) until 1:00 PM Friday.]

Please read Sections6.1 –6.5 of Burden-Faires (I assume you already know 6.4 on the determinant).

Problems

1. Sec. 6.1 #6(a,b)

2. Sec. 6.2 #10

3. Sec. 6.2 #2(a,b), #4(a,b), #6(a,b), #10(a,b)

4. Sec. 6.3 #10, #14 (this is continued in the next problem)

5. Notation: Let $M = \left(\begin{array}{c|c} \text{A} & \text{B} \\ \hline \text{C} & \text{D} \end{array}\right)$ be an $(n+k)\times(n+k)$ block matrix partitioned into the $n\times n$ matrix A, the $n\times k$ matrix B, the $k\times n$ matrix C and the $k\times k$ matrix D.

   Let $N = \left(\begin{array}{c|c} \text{W} & \text{X} \\ \hline \text{Y} & \text{Z} \end{array}\right)$ is another matrix with the same "shape" as M.

   Show that the naive matrix multiplication
   $$MN = \left(\begin{array}{c|c} \text{AW+BY} & \text{AX+BZ} \\ \hline \text{CW+DY} & \text{CX+DZ} \end{array}\right)$$
   is correct.

6. [Inverses] (use the notation of the previous problem)
   a) Show that matrices of the above form but with $C = 0$ are a sub-ring.
   b) If $C = 0$, show that M in invertible if and only if both A and D are invertible – and find a formula for M^{-1} involving A^{-1}, etc.
   c) More generally, if A is invertible, show that M is invertible if and only if the matrix $H := D - CA^{-1}B$ is invertible – in which case
   $$M^{-1} = \begin{pmatrix} A^{-1} + A^{-1}BH^{-1}CA^{-1} & -A^{-1}BH^{-1} \\ -H^{-1}CA^{-1} & H^{-1} \end{pmatrix}.$$

d) Similarly, if D is invertible, show that M is invertible if and only if the matrix $K := A - BD^{-1}C$ is invertible – in which case

$$M^{-1} = \begin{pmatrix} K^{-1} & -K^{-1}BD^{-1} \\ -D^{-1}CK^{-1} & D^{-1} + D^{-1}CK^{-1}BD^{-1} \end{pmatrix}.$$

e) For which values of a, b, and c is the following matrix invertible? What is the inverse?

$$S := \begin{pmatrix} a & b & b & \cdots & b & b \\ c & a & 0 & & 0 & 0 \\ c & 0 & a & & 0 & 0 \\ \vdots & \vdots & & \ddots & & \vdots \\ c & 0 & 0 & \cdots & a & 0 \\ c & 0 & 0 & \cdots & 0 & a \end{pmatrix}$$

f) Let the square matrix M have the block form $M := \left(\begin{smallmatrix} A & B \\ C & 0 \end{smallmatrix}\right)$, so $D = 0$. If B and C are square, show that M is invertible if and only if both B and C are invertible, and find an explicit formula for M^{-1}. [Answer: $M^{-1} := \begin{pmatrix} 0 & C^{-1} \\ B^{-1} & -B^{-1}AC^{-1} \end{pmatrix}$].

[Last revised: October 19, 2018]