Math 320, Fall 2018 **Jerry L. Kazdan**

Problem Set 4

DUE: Thurs. Sep. 27 in class. [Late papers accepted (without penalty) until 1:00 PM Friday.]

Please read Sections 3.6–3.7, 4.2–4.3, and 4.5 of Burden-Faires. [Note that Section 4.5 is really a special case of the ideas in 4.2 applied to the trapezoidal rule]

Also, look over Sections 3.6–3.7, 4.1, 4.4, 4.6 to understand the essential point of the issues discussed [imagine giving a 5 minute summary to a friend]. We will not be covering these sections.

Problems

1. [Sec. 3.5 #16] Construct a natural cubic spline to approximate $f(x) = e^{-x}$ by using the values of $f(x)$ at $x = 0$, 0.25, 0.5, 0.75, and 1.0. Integrate the spline over [0, 1] and compare the result to $\int_0^1 e^{-x}\,dx = 1 - 1/e$.

   Use the spline to compute $f'(0.5)$ and $f''(0.5)$. Compare these to their true values.

2. [Sec. 3.5 #20] Suppose the data $\{x_j,\, f(x_j)\}_{j=1}^n$ happen to lie on a straight line. What can be said about the natural and clamped cubic spline approximations to $f(x)$?

3. [Sec. 3.5 #29] A car driving along a straight road is clocked at a number of points. Here is the data (time in seconds, distance in feet, speed in ft/sec):

Time	0	3	5	8	13
Distance	0	225	383	623	993
Speed	75	77	80	74	72

   a) Use a clamped cubic spline to predict the position of the car and its speed at time $t = 10$.

   b) Use the derivatives of the spline to determine whether the car ever exceeded the speed limit of 55 m/h.

   c) What is the predicted maximum speed of the car?

4. [Sec. 4.2 #5] The following data give approximations to the integral $M = \displaystyle\int_0^\pi \sin x\,dx$:

   $$N_1(h) = 1.570796, \quad N_1(\tfrac{h}{2}) = 1.896119, \quad N_1(\tfrac{h}{4}) = 1.974232, \quad N_1(\tfrac{h}{8}) = 1.993570.$$

   Assuming $M = N_1(h) + K_1h^2 + K_2h^4 + K_3h^8 + O(h^{10})$, construct an extrapolation table to determine $N_4(h)$.

5. [Sections 4.3 & 4.5] Apply the trapezoidal rule, Simpson's rule, and Richardson/Romberg extrapolation with various step sizes h to compute:

$$\int_0^{\pi} (x^2 + \sin x)\, dx.$$

Discuss your results, including comparison with the exact value.

[Last revised: September 22, 2018]