Problem Set 3

DUE: Thurs. Sep. 20 in class. [Late papers accepted (without penalty) until 1:00 PM Friday.]

Please read Sections 2.6, 3.1, and 3.5 of Burden-Faires.

Also, look over Sections 2.5, 2.7, 3.2–3.4 to understand the essential point of the issues discussed [imagine giving a 5 minute summary to a friend]. We will not be covering these sections.

Below are some rudimentary exercises (all from the 9th Edition of Burden-Faires).

Problems

1. [Sec. 2.4 #2a] Use Newton's method to find solution(s) accurate to within 10^{-5} for

   a). $1 - 4x\cos x + 2x^2 + \cos 2X = 0$, for $0 \le x \le 1$.

2. [Sec. 2.4 #6] Show that the following sequences converge linearly to $p = 0$. How large must n be before $|p_n - p| \le 5 \times 10^{-2}$?

   a). $p_n = \dfrac{1}{n}, \quad n \ge 1$ b). $p_n = \dfrac{1}{n^2}, \quad n \ge 1$.

3. [Sec. 2.4 #8a,b)] a. Show that the sequence $p_n = 10^{-(2^n)}$ converges quadratically to 0.

   b). Show that the sequence $p_n = 10^{-(n^k)}$ does *not* converge quadratically to 0 regardless of the size of the exponent $k > 1$.

4. [Sec 2.4 #11] Show that the Bisection algorithm 2.1 gives a sequence with an error bound that converges linearly to 0.

5. [Sec. 3.1 #2a,b)] For the given functions $f(x)$, let $x_0 = 1$, $x_1 = 1.25$, and $x_2 = 1.6$. Construct interpolation polynomials of degree at most one and at most two to approximate $f(1.4)$, and find the absolute error.

   a(. $f(x) = \sin \pi x$. b). $f(x) = (x-1)^{1/3}$

6. [Sec. 3.1 #4a,b)] Use Theorem 3.3 to find an error bound for the approximations you found in the previous problem.

7. [Sec. 3.1 #9] Let $P_3(x)$ be the interpolation polynmial for the data: $(0,0)$, $(0.5, y)$, $(1, 3)$, and $(2, 2)$. If the coefficient of x^3 in $P_3(x)$ is 6, find y.

8. [Sec. 3.1 #14c)] Let $f(x) = e^x$ for $0 \leq x \leq 2$. Approximate $f(0.25)$ and $f(0.75)$ by using the second degree interpolating polynomial with $x_0 = 0$. $x_1 = 1$ and $x_2 = 2$.

9. [Sec. 3.1 #18]

   a) The Introduction to this chapter included a table listing the population of the United States from 1959 to 2000. Use Lagrange interpolation to approximate the population in the years 1940, 1975, and 2020.

   b) The population in 1940 was approximately 132,165,000. How accurate do you think your 1975 and 2000 figures are?

10. [Sec. 3.1 #20b)] In Exercise 26 of Section 1.1 a Maclaurin series was integrated to approximate $\operatorname{erf}(1)$. where $\operatorname{erf}(x)$ is the *normal distribution error function* defined by

$$\operatorname{erf}(x) = \frac{2}{\sqrt{\pi}} \int_0^x e^{-t^2}\, dt.$$

   a) Use the Maclaurin series to construct a table for $\operatorname{erf}(x)$ that is accuratee to within 10^{-4} for $\operatorname{erf}(x_i)$, where $x_i = 0.2i$ for $i = 0, 1, \ldots, 5$.

   b) Use both linear and quadratic interpolation to obtain an approximation to $\operatorname{erf}(\frac{1}{3})$. Which approach seems most feasible?

[Last revised: September 20, 2018]