Problem Set 2

DUE: Thurs. Sep. 13 in class. [Late papers accepted (without penalty) until 1:00 PM Friday.]

Please read Sections 2.1 – 2.5 of Burden-Faires

Below are lots of Exercises (all from the 9th Edition of Burden-Faires). Fortunately most of them are short.

Problems

1. [B-F p. 39 #8] This concerns computing $\sum_{k=1}^{n}\sum_{j=1}^{k} a_k b_j$.
   a) How many multiplications and additions are needed?
   b) Modify the above sum to an equivalent form that reduced the number of computations.

2. [Sec. 2.1 #1] Use the bisection method to find p_3 for $f(x) = \sqrt{x} - \cos x$ on [0, 1].

3. [Sec. 2.1 #10] Let $f(x) := (x+2)(x+1)x(x-1)(x-2)$. To which zero of f does the Bisection method converge when applied to the following intervals?

   $a)$ $[-1.5,\ 2.5]$ $b)$ $[-0.5,\ 3.4]$ $c)$ $[-0.5,\ 3]$ $d)$ $[-3,\ -0.5]$

4. [Sec. 2.1 #15] Use Theorem 2.1 to find a bound for the number of iterations needed to achieve an approximation with accuracy 10^{-4} to the solution of $x^3 + x - 4 = 0$ lying in the interval $[1, 2]$. Find an approximation to the root with this degree of accuracy.

5. [Sec 2.2 #5] Use a fixed point iteration method to determine a solution accurate to within 10^{-2} of $x^4 - 3x^2 - 3 = 0$ on $[1, 2]$. Use $p_0 = 1$.

6. [Sec 2.2 #8] Use Theorem 2.3 to show that $g(x) = 2^{-x}$ has a unique fixed point on $[\frac{1}{3}. 1]$. Use fixed point iteration to find an approximation to the fixed point accurate to within 10^{-4}. Use Corollary 2.5 to estimate the number of iterations required to achieve 10^{-4} accuracy, and compare this theoretical estimate to the number actually needed.

7. [Sec 2.2 #20a)] If A is any positive number, show that the sequence defined by

   $$x_n = \frac{1}{2}x_{n-1} + \frac{A}{2x_{n-1}}, \quad \text{for} \quad n = 1, 2, \ldots$$

   whenever $x_0 > 0$.

8. [Sec 2.2 #24] Let$g \in C^1[a,b]$ and p be in (a,b) with $g(p) = p$ and $|g'(p)| > 1$. Show that there exists a $\delta > 0$ such that if $0 < |p_0 - p| < \delta$, then $|p_0 - p| < |p_1 - p|$. Thus, no matter how close the initial approximation p_0 is to p, the next iterate, p_1, is *further* away so the fixed point iteration does not converge if $p_0 \neq p$. In this case p is called a *repulsive* fixed point.

9. [Sec. 2.3 #6a)] Use Newton's method to find a solution accurate to within 10^{-5} of

$$e^x + 2^{-x} + 2\cos x - 6 = 0 \quad \text{for} \quad 1 \leq x \leq 2.$$

10. [Sec. 2.3 #20(d,e,f)] The equation $x^2 - 10\cos x = 0$ has two solutions x_1 and x_2. Use Newton's method to approximate them to within 10^{-5} with the following values of p_0: d). $p_0 = 25$, e). $p_0 = 50$, f). $p_0 = 100$.

11. [Sec. 2.3 #24. This is the example at the beginning of Chapter 2, p. 47-48]. Find an approximation for λ, accurate to within 10^{-4} fr the population equation

$$1,564,000 = 1,000,000e^\lambda + \frac{435,000}{\lambda}(e^\lambda - 1)$$

Use this value to predict the population at the end of the second year – assuming that the immigrate ion rate this year remains at 435,000 people per year.

12. [Sec. 2.3 #31] The logistic population growth model is described by an equation of the form

$$P(t) = \frac{P_L}{1 - ce^{kt}},$$

where P_L, c, and k are positive constants. Here $P(t)$ is the population at time t, $P_L = \lim_{t\to\infty} P(t)$ is the limiting value of the population.

   a) Use the census data for 1950, 1960, and 1970 [see the table on page 105) to determine the constants P_L, c and k.

   b) Then use the equation (with $t = 0$ in 1950) to predict the population for 1980 and 2010. Compare the 1980 prediction with the actual value.

13. Compute $\sqrt{3}$ by the following methods:

   a) bisection [Sec. 2.1 #12]

   b) iteration [Sec. 2.2 #9]

   c) Newton's method [Sec. 2.3]

   d) Write a few sentences comparing your results.

[Last revised: September 9, 2018]