

Problem Set 10

DUE: Thurs. Nov. 29 in class. [Late papers accepted (without penalty) until 1:00 PM Friday.]

Chapters 7.1-7.3. 7.5 in Burden-Faires

Problems

1. Let $A := \begin{pmatrix} 4 & -3 & 2 \\ -1 & 0 & 5 \\ 2 & 6 & -2 \end{pmatrix}$. Among all vectors x satisfying $\|x\|_\infty \leq 1$, find one for which $\|Ax\|_\infty$ is as large as possible. Also, what is the numerical value of $\|A\|_\infty$?

2. Using the norm $\|A\|_1$, calculate the condition number of the matrices

$$A = \begin{pmatrix} 1 & 1 + \epsilon \\ 1 - \epsilon & 1 \end{pmatrix} \quad \text{and} \quad B = \begin{pmatrix} 7 & 8 \\ 9 & 10 \end{pmatrix}$$

3. For each of the norms $\|A\|_1$, $\|A\|_2$, $\|A\|_\infty$, compute the condition numbers of the matrices

$$A = \begin{pmatrix} a+1 & a \\ a & a-1 \end{pmatrix}, \quad B = \begin{pmatrix} 0 & 1 \\ -2 & 0 \end{pmatrix}, \quad C = \begin{pmatrix} 0 & 1 \\ 1 & 1 \end{pmatrix}.$$

4. a) Show that the condition number for an invertible matrix must be at least 1.
b) Which matrices have condition number 1?

5. In a normed linear space, show that $\|x + y\| \geq | \|x\| - \|y\| |$.

6. In solving the system of equations $Ax = b$ with $A = \begin{pmatrix} 1 & 2 \\ 1 & 2.01 \end{pmatrix}$ predict how slight changes in b will effect the solution x . Test your prediction in the concrete cases where $b = (4, 4)$ and $\hat{b} = (3, 5)$.

7. a) If a square matrix A satisfies an inequality $\|Ax\| \geq c\|x\|$ for all x , with $c > 0$, show that A is invertible and $\|A^{-1}\| \leq 1/c$. [This is true for any vector norm and its corresponding matrix norm.]
b) Prove that if A is (row) diagonally dominant, then it will have the above property. Give a value for c when the norm $\|\cdot\|_\infty$ is used.

8. Let $B = I - A$ where $A = \begin{pmatrix} 0.1 & 0.2 & 0.3 \\ -0.1 & 0.0 & 0.1 \\ -0.3 & -0.2 & -0.1 \end{pmatrix}$ Use the first ten terms of the series $(I - A)^{-1} = I + A + A^2 + A^3 + \dots$ to compute B^{-1}

9. Burden-Faires, Sec. 7.3 #2c)

10. Burden-Faires, Sec. 7.3 #13

11. Burden-Faires, Sec. 7.3 #16

12. Burden-Faires, Sec. 7.5 #2b

13. Burden-Faires, Sec. 7.5 #4b

[Last revised: November 21, 2018]