

### Problem Set 1

Please read Chapter 1 of Burden-Faires, particularly Sections 1.1, 1.2, and 1.3. Section 1.4 is both interesting and important but we will not cover it.

The following short answer problems from Burden-Faires [9th Edition] are not assigned, but you should know how to do them. Some short answer problems will be on our exams.

p. 15-16 #2, 10,    p. 28-30, #1a,b,c #2, #4, #10    p. 30 #6, 7

#### Problems

DUE: Thurs. Sep. 6 in class. [Late papers accepted (without penalty) until 1:00 PM Friday.]

1. Let  $c := 271.4$  base 10. Write  $c$  in both base 2 and base 7.

2. [B-F p. 30 #8] This concerns computing  $\sum_{k=1}^n \sum_{j=1}^k a_k b_j$ .

- How many multiplications and additions are needed?
- Modify the above sum to an equivalent form that reduced the number of computations.

3. [Burden-Faires p. 30 #18] The Taylor polynomial of degree  $n$  for  $e^x$  is  $e^x \approx \sum_0^n x^k/k!$ .

a) Use the Taylor polynomial of degree 9 and three digit chopping arithmetic to find an approximation to  $e^{-5}$  by each of the following methods:

$$\text{i). } e^{-5} \approx \sum_0^9 \frac{(-5)^k}{k!} \qquad \text{ii). } e^{-5} = \frac{1}{e^5} = \frac{1}{\sum_0^9 \frac{5^k}{k!}}$$

b) A (correct) three digit approximation is:  $e^{-5} \approx 6.74 \times 10^{-3}$ . Explain any discrepancies between this and your computations above.

4. [Burden-Faires p. 30 #22] A rectangular parallelepiped has sides of length 3cm, 4cm, and 5cm (measured to the nearest centimeter). What are the best upper and lower bounds to the volume? What are the best upper and lower bounds to the surface area?

5. [B-F p. 39 #1a] Use three-digit chopping arithmetic to compute  $\sum_1^{10} (1/k^2)$  in two different ways:  $\frac{1}{1} + \frac{1}{4} + \dots + \frac{1}{100}$  and  $\frac{1}{100} + \dots + \frac{1}{1}$ . Which method is more accurate? Why?

6. [COST OF MULTIPLYING MATRICES]. Let  $A$  and  $B$  be real  $n \times n$  matrices. If you compute the product,  $AB$ , how many multiplications will you use? [Justify your assertion.]

7. [COST OF COMPUTING THE DETERMINANT.] If  $A$  is an  $n \times n$  real matrix, let  $\text{Mult}(n)$  be the number of multiplications needed to compute the determinant of  $A$  (additions and subtractions are “free”). Thus  $\text{Mult}(2) = 2$  and  $\text{Mult}(3) = 9$ . Compute  $\text{Mult}(n)$  for  $n = 4, 5, 10$ .
8. Let  $S_N = 1 + \frac{1}{2} + \frac{1}{3} + \cdots + \frac{1}{N}$ . For both of the following, justify your reasoning.
- Write a short program that computes  $S_{10,000}$  and have the program print how much time the computation took.
  - Find an integer  $N$  so that  $S_N > 100$ . [Use the idea behind the *integral test* for the convergence of series.]
9. [ZEROS OF POLYNOMIALS] Let  $p(z) := z^n + a_{n-1}z^{n-1} + \cdots + a_1z + a_0$ , where the coefficients  $a_k$  may be complex numbers. Find a disk  $D := \{z \in \mathbb{C} \mid |z| \leq R\}$  so that all of the (possibly complex) zeroes of  $p$  lie in this disk. Of course the radius  $R$  depends on the coefficients.

SUGGESTION: First do the cases of quadratic and cubic polynomials – *without* using the explicit formulas for the roots. For  $n = 2$  one approach begins with the observation that if  $z \neq 0$  is a root then

$$z = -\left(a_1 + \frac{a_0}{z}\right)$$

so if  $|z| \geq 1$  then

$$|z| \leq |a_1| + \left|\frac{a_0}{z}\right| \leq |a_1| + |a_0|.$$

This approach generalizes.

### Bonus Problem

[Submit these directly to Professor Kazdan (same Due Date).]

B-1 Consider solutions of the second order linear difference equation

$$x_{n+2} + bx_{n+1} + cx_n = 0, \tag{1}$$

where  $b$  and  $c$  are real constants. Clearly if one is given the *initial conditions*  $x_0$  and  $x_1$  one can compute  $x_2$ , and then use this to compute  $x_3$  etc. But there is a more systematic way to find a formula for  $x_n$  directly.

Note that this equation is *linear*, that is, a constant times a solution is also a solution and the sum of two solutions is a solution.

- a) [Example (Fibonacci):  $x_{n+2} = x_{n+1} + x_n$  with *initial conditions*  $x_0 = 0$  and  $x_1 = 1$ ]. Clearly  $x_2 = 1$  and  $x_3 = 2$  etc.

PROCEDURE: Seek a special solution in the form  $x_n = r^n$ . Find the two values,  $r_1$  and  $r_2$  of  $r$  for this  $x_n$  to be a solution. Then by linearity, for any constants  $A$  and  $B$ ,

$x_n = Ar_1^n + Br_2^n$  is also a solution. Then use the given initial value we can compute the constants  $A$  and  $B$ .

Carry this out for this example.

- b) Show that the same procedure works for equation (1) if one assumes that  $b^2 - 4c > 0$ .

[Last revised: September 10, 2018]