Ivy League Basketball 2002

Notes by Prof. G. Porter.

At the conclusion of the regular (before playoffs) 2002 Ivy League Basketball season there were three teams tied for first place. The standings were as follows:

Pennsylvania	11 – 3
Yale	11 – 3
Princeton	11 – 3
Brown	8 – 6
Harvard	7 – 7
Columbia	4 – 10
Dartmouth	2 – 12
Cornell	2 – 12

Because only one team would qualify for the NCAA tournament there were two playoff games to determine the team that would go to the tournament. An alternative would have been to use Linear Algebra to rank the teams. In particular we may assume that the final ranking of a team is somehow related to the ranking of the teams that it has defeated during the regular season. Each team played every other team twice during the season so let a_{ij} be the number of times that team i defeated team j and let R_j be the (unknown) rank of team j. It is reasonable that the rank of a team is proportional to the weighted sum of its win record against the other teams:

$$R_i = c\sum_{j=1}^{8} a_{ij}R_j, \qquad \text{that is,} \qquad R = cAR.$$

Here c is the proportionality constant. Writing $c = 1/\lambda$, we recognize this as an eigenvalue problem, $AR = \lambda R$. By the Perron-Frobenius Theorem, the matrix A has a unique largest eigenvalue, it is real, and associated with that there is a real eigenvector, all of whose terms are positive. The other eigenvalues (possibly complex) all have absolute value less than this largest one. We can use this eigenvector to rank the basketball teams. The matrix A is given below. Note that the columns have the same labels (names of colleges) as the rows.

$$\begin{pmatrix} & \text{Pe} & \text{Ya} & \text{Pr} & \text{Ha} & \text{Br} & \text{Da} & \text{Cl} & \text{Cr} \\ \text{Pennsylvania} & 0 & 1 & 2 & 1 & 2 & 2 & 1 & 2 \\ \text{Yale} & 1 & 0 & 1 & 2 & 1 & 2 & 2 & 2 \\ \text{Princeton} & 0 & 1 & 0 & 2 & 2 & 2 & 2 & 2 \\ \text{Harvard} & 1 & 0 & 0 & 0 & 1 & 2 & 2 & 1 \\ \text{Brown} & 0 & 1 & 0 & 1 & 0 & 2 & 2 & 2 \\ \text{Dartmouth} & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 \\ \text{Columbia} & 1 & 0 & 0 & 0 & 0 & 1 & 0 & 2 \\ \text{Cornell} & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 \end{pmatrix}$$

so

$$A = \begin{pmatrix} 0 & 1 & 2 & 1 & 2 & 2 & 1 & 2 \\ 1 & 0 & 1 & 2 & 1 & 2 & 2 & 2 \\ 0 & 1 & 0 & 2 & 2 & 2 & 2 & 2 \\ 1 & 0 & 0 & 0 & 1 & 2 & 2 & 1 \\ 0 & 1 & 0 & 1 & 0 & 2 & 2 & 2 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 \\ 1 & 0 & 0 & 0 & 0 & 1 & 0 & 2 \\ 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 \end{pmatrix}$$

The eigenvalues of A are

$$\begin{array}{c} 4.7 \\ 0.58 \\ -0.76 + 1.78i \\ -0.76 - 1.78i \\ -0.93 + 1.19i \\ -0.93 - 1.19i \\ -0.97 + 0.47i \\ -0.97 - 0.47i \end{array}$$

and the eigenvector corresponding to the largest eigenvalue is

$$R = \begin{pmatrix} 0.57 \\ 0.52 \\ 0.47 \\ 0.29 \\ 0.29 \\ 0.048 \\ 0.16 \\ 0.07 \end{pmatrix}.$$

If we had used this procedure the final ranking would have been

$$\begin{pmatrix} \text{Pennsylvania} & .57 \\ \text{Yale} & .52 \\ \text{Princeton} & .47 \\ \text{Brown} & .29 \\ \text{Harvard} & .28 \\ \text{Columbia} & .16 \\ \text{Cornell} & .070 \\ \text{Dartmouth} & .048 \end{pmatrix}$$

The actual rankings after the playoffs was:

Pennsylvania	12 – 3
Yale	12 – 4
Princeton	11 – 4
Brown	8 – 6
Harvard	7 – 7
Columbia	4 – 10
Dartmouth	2 – 12
Cornell	2 – 12

Additional Reference: Some notes "Searching the web with eigenvectors," by H. Wilf.
`http://www.math.upenn.edu/~wilf/website/KendallWei.pdf`