Problem Set 1

DUE: In class Thursday, Jan. 17. *Late papers will be accepted until 1:00 PM Friday.*

These problems are intended to be straightforward with not much computation.

1. Solve all of the following equations. [Note that the left sides of these equations are identical.]

   a). $\begin{aligned} 2x+5y &= 5 \\ x+3y &= -1 \end{aligned}$ b). $\begin{aligned} 2x+5y &= 0 \\ x+3y &= -2 \end{aligned}$ c). $\begin{aligned} 2x+5y &= 1 \\ x+3y &= 0 \end{aligned}$ d). $\begin{aligned} 2x+5y &= 2 \\ x+3y &= 1 \end{aligned}$

2. [Bretscher, Sec.2.1 #13] Finding the inverse of a matrix A means solving the system of equations $A\vec{x}=\vec{y}$ for $\vec{x}$, so $\vec{x}=A^{-1}\vec{y}$.
   a) Let $A := \left(\begin{smallmatrix} 1 & 2 \\ c & 6 \end{smallmatrix}\right)$. With your bare hands (as on page 2 of the textbook – not using anything about determinants) show that A is invertible if and only if $c \neq 3$.
   b) Let $M := \left(\begin{smallmatrix} a & b \\ c & d \end{smallmatrix}\right)$. With your bare hands (not using anything about determinants) show that M is invertible if and only if $ad-bc \neq 0$. [*Hint:* Treat the cases $a \neq 0$ and $a = 0$ separately.]

3. Let A and B be 2×2 matrices.
   a) If B is invertible and $AB = 0$, show that $A = 0$.
   b) Give an example where $AB = 0$ but $BA \neq 0$.
   c) Find an example of a 2×2 matrix with the property that $A^2 = 0$ but $A \neq 0$.
   d) Find all *invertible* $n\times n$ matrices A with the property $A^2 = 3A$.

4. [Bretscher, Sec.2.3 #19] Find all the matrices that commute with $A := \left(\begin{smallmatrix} 0 & -2 \\ 2 & 0 \end{smallmatrix}\right)$.

5. a) Find a real 2×2 matrix A (other than $A = \pm I$) such that $A^2 = I$.
   b) Find a real 2×2 matrix A such that $A^4 = I$ but $A^2 \neq I$.

6. Let L, M, and P be linear maps from the (x_1, x_2) plane to the (y_1, y_2) plane:

   L is rotation by 90 degrees counterclockwise.

   M is reflection across the line $x_1 = x_2$.

   $N\vec{v} := -\vec{v}$ for any vector $\vec{v} \in \mathbb{R}^2$.

   a) Find matrices representing each of the linear maps L, M, and N.
   b) Draw pictures describing the actions of the maps L, M, and N and the compositions: LM, ML, LN, NL, MN, and NM.
   c) Which pairs of these maps commute?

d) Which of the following identities are correct—and why?

1)	$L^2 = N$	2)	$N^2 = I$	3)	$L^4 = I$	4)	$L^5 = L$
5)	$M^2 = I$	6)	$M^3 = M$	7)	$MNM = N$	8)	$NMN = L$

1[Last revised: May 5, 2013]