Math 312, Homework 7 (due Friday, November 2nd)

Name:________________________________ (if you choose to use this as a coversheet)

Reading Chapter 7 of Bretscher (in section 7.1, only begin at the bottom of pg. 299)

Book problems

- Section 7.1, problems 1, 2, 3, 6, 8, 16
- Section 7.2, problems 2, 8, 12, 15, 24 (note the columns sum to 1), 32
- Section 7.3, problems 2, 8, 12, 18, 44
- Section 7.4, problems 2, 12, 19, 32

Additional Problems

1. Suppose λ is an eigenvalue of an $n \times n$ matrix A. Prove that the λ-eigenspace (call it E_λ) is a subspace of $\mathbb{R}^n$.

2. In class, we stated the theorem: "If A is $n \times n$ and has n **distinct real eigenvalues**, then A has n linearly independent eigenvectors," and proved the case for $n = 2$. Using a similar idea, prove the theorem for $n = 3$ (and notice how your proof would work for any n).

3. Suppose a matrix A has characteristic polynomial $\lambda^2(\lambda - 1)(\lambda + 2)^3$.

   (a) Describe the eigenvalues of A and their (algebraic) multiplicities.

   (b) What are the fewest and greatest number of linearly independent eigenvectors that A could have? Why?

   (c) Is A invertible, or is there not enough information to tell? Why?

   (d) What are the possible values of the rank of A? Why?

   (e) Under what circumstances is A diagonalizable?

4. For which of the following linear transformations T of $\mathbb{R}^2$ is it possible to find a basis $\mathcal{B}$ for which $[T]_{\mathcal{B}}$ is diagonal? Explain.

   (a) A rotation by $\pi/4$ radians clockwise.

   (b) A rotation by π radians counter-clockwise.

   (c) A reflection across the line $y = -x$.

   (d) A skew that sends $\vec{e}_1$ to itself and $\vec{e}_2$ to $-\vec{e}_1 + \vec{e}_2$.