Math 312, Homework 5 (due Friday, October 12th)

Name:________________________________ (if you choose to use this as a coversheet)

Reading

- Read chapter 4 of Bretscher

Book problems NOTE: you can ignore the "GOAL" paragraph at the start of each exercise set in Bretscher – this is not a set of instructions for particular problems.

- Section 4.1, problems 48, 50
- Section 4.2, problems 1 (note I_2 is the 2×2 identity), 6, 24, 25, 37, 43, 64 (Hint on 43: what can you say about a degree-two polynomial with three roots?)
- Section 4.3, problems 2, 12, 15, 21, 25, 32

Additonal problems

1. Let $V = M_{3,3}(\mathbb{R})$, the vector space of $n \times n$ matrices with real entries. Define a transformation $T : V \to V$ where $T(A) = \frac{1}{2}(A + A^T)$. (Here, A^T is the matrix transpose of A.)
   (a) Verify that T is linear. You may use familiar facts about transpose.
   (b) Describe the image of T, and find its dimension.
   (c) Describe the kernel of T, and find its dimension.
   (d) Verify the rank and nullity add up what you would expect. (Final note: T is called the symmetrization operator.)
2. Prove that $\mathbb{R}^n$ is not isomorphic to $\mathbb{R}^m$ if $n \neq m$. Remember, a linear transformation is an isomorphism if it is both one-to-one and onto. Hint: handle the cases $n > m$ and $n < m$ separately, and use a well-known theorem.
3. Let $T : \mathbb{P}_2 \to \mathbb{P}_2$ be the transformation that sends $p(t)$ to its antiderivative, divided by t. (Here, the "$+C$" constant is chosen to be zero in the antiderivative). For instance, T sends $t^2 + 2t - 1$ to $\frac{1}{3}t^2 + t - 1$.
   (a) Prove that T is a *linear* transformation.
   (b) Find the kernel of T, and find its dimension.
   (c) Find the range of T, and find its dimension.
   (d) Verify the dimension of the kernel and the dimension of the range add up to what you would expect.
   (e) Using the standard basis $\{t^2, t, 1\}$ for $\mathbb{P}_2$, represent the linear transformation T as a matrix A.
   (f) Using your matrix represention from (e), find $T(t - 2)$.