Math 312, Homework 4 (due Friday, October 5th)

Name:________________________________ (if you choose to use this as a coversheet)

Reading

- Read sections 3.4 and 4.1 of Bretscher. As needed, review sections 3.1 – 3.3 and the additional material posted on blackboard.

Book problems NOTE: you can ignore the "GOAL" paragraph at the start of each exercise set in Bretscher – this is not a set of instructions for particular problems.

- Section 3.4, problems 2, 4, 7, 12, 19*, 22*, 27, 32, 45, 58 (* in 19 and 22, ignore the suggestions a,b,c; proceed however you wish)
- Section 4.1, problems (please be sure to explain your answers) 1, 2, 4, 7, 10, 20, 29.

Additonal problems

1. Are the following collections vector spaces? Explain. If so, find bases and compute the dimension.
   (a) $\mathbb{C}$, the set of complex numbers
   (b) the set of 3×3 symmetric matrices, A (meaning A equals its own transpose)
   (c) the set of polynomials p of degree at most two such that $p(1) = 0$
   (d) the set of solutions f to the ODE:
   $$f'' + 2f' - 10f = 0.$$
   (e) the set of 2×2 matrices that commute with $B = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$.
2. For the $(7, 4)$ Hamming code, consider the code generator matrix G and the parity check matrix P.
   (a) Without doing any computations, explain why PG is the zero matrix (using arithmetic mod 2). Hint: what happens to any vector $\vec{x} \in \mathbb{R}$ with binary entries when you apply G followed by P?
   (b) Suppose the sender wants to send the 4 bits `[1 1 0 1]`. What 7-bit vector should they send if they use the $(7, 4)$ Hamming code?
   (c) In another scenario, suppose the receiver gets the message `[0 0 1 0 0 1 0 ]`. Did an error occur? What is the original 4 bit message?
3. Using our discussion on computer graphics, consider the linear transformation T of $\mathbb{R}^3$ that first rotates about the x-axis by $\pi/2$ (counter-clockwise from y to z), then $\pi/2$ about the z-axis (counter-clockwise from x to y). (OVER)

(a) By looking at where T sends the standard basis vectors, find the standard matrix representation of T. (You might check your answer using matrix multiplication.)

(b) How many times must you apply T to obtain the identity?