Math 312, Homework 2 (due Friday, Sep. 21)

Name:______________________________ (if you choose to use this as a coversheet)

Reading

- Read chapter 2 of Bretscher; also pp. 106–109

Book problems

- Section 2.1, problems 28, 30, 44
- Section 2.2, problems 4, 20, 22, 32
- Section 2.3, problems 18, 38. Hint on 18: Let $B = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ and set up the equation $AB = BA$. (Also: if you're rusty on matrix multiplication, please practice with problems 1–14 as needed.)
- Section 2.4, problems 12, 20, 44, 76. (Also: if you're rusty on finding matrix inverses, please practice with problems 1–15 as needed.)
- Section 3.1, problems 7, 9

Additonal problems

1. Let $T : \mathbb{R}^n \to \mathbb{R}^m$ be a linear transformation. Prove that T always sends the zero vector in $\mathbb{R}^n$ to the zero vector in $\mathbb{R}^m$. (Hint: Whatever $T(\vec{0})$ is, call it $\vec{w}$. Compute $T(\vec{0} + \vec{0})$ in two different ways to conclude that $\vec{w}$ is the zero vector.)
2. Let T be a linear transformation. Explain carefully why

$$T(c_1\vec{v}_1 + c_2\vec{v}_2 + c_3\vec{v}_3) = c_1T(\vec{v}_1) + c_2T(\vec{v}_2) + c_3T(\vec{v}_n).$$

   (The same statement holds of course for n vectors and n scalars as well.)

3. Let S and T be linear transformations from $\mathbb{R}^2$ to $\mathbb{R}^2$. Suppose S sends $\vec{e}_1$ to itself and sends $\vec{e}_2$ to $\frac{1}{2}\vec{e}_1 + \vec{e}_2$. Suppose T reflects vectors about the line $y = -x$.
   a. Find the matrix A that represents S and the matrix B that represents T.
   b. Draw pictures indicating where S and T each send the letter "F", like we did in class.
   c. Draw pictures indicating where $S \circ T$ and $T \circ S$ each send the letter "F".
4. Find a linear transformation $T : \mathbb{R}^2 \to \mathbb{R}^2$ such that $T \circ T$ is the zero transformation (i.e., sends every vector to $\vec{0}$). You're not allowed to let T itself be the zero transformation!
5. Let A and B be $n \times n$ matrices. What's wrong with the formula $(A + B)^2 = A^2 + 2AB + B^2$? Prove that if this equation is true for matrices A and B, then A and B commute.