Math 312, Fall 2012 Jerry L. Kazdan

Problem Set 6

DUE: In class Thursday, Oct. 25 *Late papers will be accepted until 1:00 PM Friday.*

REMARK: We have completed Chapter 5, Sections 5.1, 5.2, 5.3, and 5.4 (except for the QR Factorization – which we will skip). Since Fall Break interrupts this week, this assignment will be shorter.

1. [BRETSCHER, SEC. 5.2 #34] Find an orthonormal basis for the kernel of the matrix

$$A := \begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & 2 & 3 & 4 \end{pmatrix}.$$

2. [BRETSCHER, SEC. 5.4 #20] Using pencil and paper, find the least-squares solution to $A\vec{x} = \vec{b}$ where

$$A = \begin{pmatrix} 1 & 1 \\ 1 & 0 \\ 0 & 1 \end{pmatrix} \qquad \text{and} \qquad \vec{b} = \begin{pmatrix} 3 \\ 3 \\ 3 \end{pmatrix}.$$

3. Use the Method of Least Squares to find the parabola $y = ax^2 + b$ that best fits the following data given by the following four points (x_j, y_j), $j = 1, \ldots, 4$:

$$(-2, 4), \qquad (-1, 3), \qquad (0, 1), \qquad (2, 0).$$

   Ideally, you'd like to pick the coefficients a and b so that the four equations $ax_j^2 + b = y_j$, $j = 1, \ldots, 4$ are all satisfied. Since this probably can't be done, one uses least squares to find the best possible a and b.

4. The water level in the North Sea is mainly determined by the so-called M2 tide, whose period is about 12 hours. The height $H(t)$ thus roughly has the form

$$H(t) = c + a\sin(2\pi t/12) + b\cos(2\pi t/12),$$

   where time t is measured in hours (note $\sin(2\pi t/12$ and $\cos(2\pi t/12)$ are periodic with period 12 hours). Say one has the following measurements:

t (hours)	0	2	4	6	8	10
$H(t)$ (meters)	1.0	1.6	1.4	0.6	0.2	0.8

   Use the method of least squares to find the constants a, b, and c in $H(t)$ for this data.

5. Let A be a real matrix, not necessarily square.
   a) Show that both A^*A and AA^* are self-adjoint.
   b) Show that $\ker A = \ker A^*A$.[HINT: Show separately that $\ker A \subset \ker A^*A$ and $\ker A \supset \ker A^*A$. The identity $\langle \vec{x}, A^*A\vec{x} \rangle = \langle A\vec{x}, A\vec{x} \rangle$ is useful.]

[Last revised: October 25, 2012]