Math 312, Homework 10 (due Friday, November 30th)

Name:________________________________ (if you choose to use this as a coversheet)

Reading Chapter 5 of Bretscher.

Book problems

- Section 5.3, problems 30 (no illustration necessary), 31, 44,
- Section 5.4, problem 32. (For extra practice, you may wish to do #26)
- Section 5.5, problems 10, 16, 24

Additional Problems Recall that an $m \times n$ matrix A is orthogonal if $A^T A = I_n$ and a square matrix P is a projection matrix if $P^2 = P$.

1. What are the possible values of the determinant of a projection matrix? What about the possible values of the eigenvalues?

2. Recall that the matrix of orthogonal projection onto the image of A is given by $P = A(A^T A)^{-1} A^T$. Verify that P is actually a projection matrix.

3. Let $\vec{x}(t)$ be a function taking values in $\mathbb{R}^n$ that satisfies the ODE
$$\frac{d}{dt}\vec{x}(t) = A\vec{x}(t),$$
where A is a skew-symmetric $n \times n$ matrix. Prove that the length of $\vec{x}(t)$ is independent of t. (Hint: differentiate the length-squared, and use the last problem from last week's assignment.)

4. Verify that the L^2 inner product (defined on the vector space of continuous functions on an interval $[a, b]$) is actually an inner product. (You may use familiar facts from calculus.)

5. Explain carefully why a matrix with more columns than rows can never be orthogonal.

6. Let $V \subset \mathbb{R}^3$ be the subspace spanned by $(1, 2, 3)$ and $(1, 0, -1)$. Find the vector in V closest to the vector $(2, 1, 1)$. How far is this vector from W?

7. Let A, B, C be $n \times n$ matrices, and let $\langle , \rangle$ be the usual dot product on $\mathbb{R}^n$.
   (a) Prove that if $\langle x, Cy \rangle = 0$ for all $x, y \in \mathbb{R}^n$, then C must be the zero matrix.
   (b) Prove that if $\langle x, Ay \rangle = \langle x, By \rangle$ for all $x, y \in \mathbb{R}^n$, then $A = B$. (Hint: use the previous part!)

8. Let $f : [0, 2\pi]$ be the (discontinuous) function that equals 1 on $[0, \pi)$ and equals 0 on $[\pi, 2\pi]$. Find all Fourier coefficients of f. (Hint: you'll need to evaluate the integrals in pieces.)

   Using software (such as Wolfram Alpha or a graphing calculator), graph the terms of the Fourier series of f up through $\sin(5x)$ and $\cos(5x)$.

9. Let $f : [0, 2\pi]$ be the function $f(x) = x^2$. Find all Fourier coefficients of f. (Hint: you'll need to integrate by parts.)

   Using software (such as Wolfram Alpha or a graphing calculator), graph the terms of the Fourier series of f up through $\sin(5x)$ and $\cos(5x)$.