$$u_x + 3u_y = 0$$

This example is similar to Problem Set 7 # 3a).

EXAMPLE: Find a function $u(x, y)$ that satisfies $u_x + 3u_y = 0$ with $u(0, y) = 1 + e^{2y}$.

SOLUTION:The differential equation can be written $\nabla u \cdot V = 0$ where $V = (1, 3)$. It means that at every point the directional derivative in the direction of V is 0 so $u(x, y)$ is constant along these parallel straight lines, which have the form $y = 3x + C$. Given a point (x, y) one computes $y - 3x$ to determine C, that is, which line you are on.

Thus the solution $u(x, y) = h(y - 3x)$ for some as yet unknown function $h(s)$. Now we use the initial condition $u(0, y) = 1 + e^{2y}$. It gives us

$$1 + e^{2y} = u(0, y) = h(y).$$

Consequently $\boxed{u(x, y) = 1 + e^{2(y-3x)}}$.

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