Problem Set 7

DUE: Thurs. March 19 in class. [Late papers will be accepted until 1:00 PM Friday.]

This week. Please read Chapter 7, Sections 7.1 – 7.7 in the Haberman text.

1. p. 162 #5.3.3 [First try the special case $\alpha(x) = 2x$.]

2. p. 163 #5.3.8 (& p. 188 #5.6.2)

3. p. 163 #5.3.9

4. p. 166 #5.4.1

5. p. 174 #5.5.1(a-f)

6. p. 175 #5.5.2

7. p. 188 #5.6.1(a)

8. Let $A = \begin{pmatrix} 2 & 1 & 0 \\ 1 & 2 & 0 \\ 0 & 0 & 3 \end{pmatrix}$. Note that A is symmetric.

   a) Find the eigenvalues λ_j, $j = 1, 2, 3$ and corresponding *orthogonal* eigenvectors $\vec{v}_j$.

   b) Use this to find the general solution of the homogeneous equation

   $$\frac{d\vec{x}}{dt} = A\vec{x}. \tag{1}$$

   SUGGESTION: Seek the solution as a linear combination of the eigenvectors

   $$\vec{x}(t) = \phi_1(t)\vec{v}_1 + \phi_2(t)\vec{v}_2 + \phi_3(t)\vec{v}_3, \tag{2}$$

   where the coefficients $\phi_j(t)$ are found as follows:
   (i). Use equation (2) to compute both $\frac{d\vec{x}}{dt}$ and $A\vec{x}$. Then, for equation (1) equate these to deduce that

   $$\frac{d\phi_1(t)}{dt} = \lambda_1\phi_1(t), \qquad \frac{d\phi_2(t)}{dt} = ?, \qquad \frac{d\phi_3(t)}{dt} = ?$$

   (ii). Now solve these equations for $\phi_1(t)$, $\phi_2(t)$, and $\phi_3(t)$. Then equation (2) gives the desired solution.

c) Find the solution of (1) that also satisfies the initial condition $\vec{x}(0) = (1, 0, 0)$

9. [Ignore. It repeated #7 above]

10. p. 272 #7.2.2 [correction of typo 2.7.2]

11. p. 279 #7.3.4(a)

[Last revised: March 16, 2015]