Problem Set 3

DUE: Thurs. Feb. 5 in class. [Late papers will be accepted until 1:00 PM Friday.]

This week. Please re-read all of Chapter 2 and read Chapter 4.4 in the Haberman text.

1. p. 51 #2.3.3 (b, d)
2. p. 52 #2.3.4
3. p. 65 # 2.4.1 (a, b)
4. p. 66 # 2.4.2
5. p. 66 # 2.4.6
6. p. 81 # 2.5.1 (a,b)
7. p. 81 # 2.5.2
8. p. 82 # 2.5.3
9. p. 82 # 2.5.5 (b)
10. p. 82 # 2.5.7 (a)
11. This is now Bonus -3.

Bonus Problems

[Please give this directly to Professor Kazdan]

B-1 p. 82 # 2.5.4

B-2 Let $u(x,t)$ be a solution of the heat equation $u_t = u_{xx}$ for $0 \le x \le \pi$, Say the initial and boundary conditions are

$$u(x,0) = f(x), \qquad u(0,t) = 0, \qquad u(\pi,t) = 0$$

and assume that $|f(x)| \leq M$ for $0 \leq x \leq \pi$. Then the solution this is

$$u(x,t) = \sum_{n=1}^{\infty} b_n \sin nx \; e^{-n^2 t},$$

where the $b_n = \frac{2}{\pi} \int_0^{\pi} f(x) \sin nx \, dx$ are the Fourier coefficients of f.

Say we approximate this solution only by using the first ($n = 1$) term. The question is to estimate the error in doing so.

a) Show that

$$\int_0^{\pi} |\sin nx| \, dx = 2$$

and use this to show that

$$|b_n| \leq \frac{4M}{\pi}.$$

b) Estimate the terms beginning with $n = 2$ by showing that

$$\left| \sum_{n=2}^{\infty} b_n \sin nx \; e^{-n^2 t} \right| \leq \frac{4M}{\pi} \sum_{n=2}^{\infty} e^{-n^2 t}.$$

c) Assume the first term is roughly Me^{-t}. Show that the ratio between the above error and the first term is roughly

$$\frac{4}{\pi} \sum_{n=2}^{\infty} e^{(1-n^2)t}$$

and give a numerical estimate of this at $t = 1$.

B-3 p. 82 # 2.5.9 (b)

[See `http://www.math.upenn.edu/~kazdan/241S15/hw/hw3-2.5.9b.pdf`]

[Last revised: February 4, 2015]