Problem Set 10

Due: Thurs. April 16 in class. [Late papers will be accepted until 1:00 PM Friday.]

This week: Please read Chapter 9 Sections 9.1, 9.2, 9.3, 9.5) in the Haberman text.

1. Haberman, p. 308 #7.7.2(b)

2. Find all solutions of the wave equation (in $\mathbb{R}^2$) of the form $u = e^{i\omega t} f(r)$ that are finite at the origin. Here $r = \sqrt{x^2 + y^2}$ and ω is a real constant.

3. Haberman p. 339 # 7.10.6

4. Solve the diffusion equation $u_t = \Delta u$ in a disk of radius a in the plane with $u = C$ (a constant) on the boundary and the initial condition is 0. Note that the answer is radial.

   [Suggestion: Similarly to the procedure in Section 8.2 (page 341), the first step is to reduce to a problem where the constant temperature on the boundary is zero. Thus let $w(r, \theta, t) = u(r, \theta, t) - C$. Then w also satisfies the diffusion equation – but with boundary condition $w(a, \theta, t) = 0$ and initial condition $w(r, \theta, 0) = -C$.

   Since the data does not involve the angle θ, this is now similar to the symmetric case (where w depends only on r and t) discussed in section 7.7.9 except that here the equation corresponding to (7.7.50) one only has the first derivative of w with respect to t and in eq (7.7.52) one only prescribes $w(r, 0)$.]

5. Haberman p. 346 #8.2.1(f)

6. Haberman p. 346 #8.2.2(b)

7. Haberman p. 352 #8.3.2

8. Haberman p. 346 #8.3.6

[Last revised: April 11, 2015]