Math/Philosophy 220, Law 520 Fall 2012

Problem Set 1

DUE: In class Thursday, Sept. 20. *Late papers will be accepted until 1:00 PM Friday.*

REMARK: Suggestion for #4b).

4. The number e is defined as $e = 1 + 1 + \frac{1}{2!} + \frac{1}{3!} + \frac{1}{4!} + \cdots$. Prove that e is not a rational number by the following steps.

a) Show that $2 < e < 3$. So e is definitely not an integer.

b) By contradiction, say $e = \frac{p}{q}$, where p and q are positive integers with $q \geq 2$. Show that

$$e\,q! = N + \frac{c}{q+1},$$

where N is an integer and $0 < c < e$. Thus, conclude that $\frac{c}{q+1}$ must be an integer.

c) Then show that this contradicts $e < 3$ and $q + 1 \geq 3$.

Hint for part b)

$$\begin{aligned} e\,q! =& q!\left[1 + 1 + \frac{1}{2!} + \frac{1}{3!} + \cdots + \frac{1}{q!}\right] + q!\left[\frac{1}{(q+1)!} + \frac{1}{(q+2)!} + \frac{1}{(q+3)!} + \frac{1}{(q+4)!} + \cdots\right] \\ =& N + \frac{1}{q+1}\left[1 + \frac{1}{q+2} + \frac{1}{(q+2)(q+3)} + \frac{1}{(q+2)(q+3)(q+4)} + \cdots\right] \end{aligned}$$

Let

$$c := 1 + \frac{1}{q+2} + \frac{1}{(q+2)(q+3)} + \frac{1}{(q+2)(q+3)(q+4)} + \cdots$$

so you are asked to show that $1 < c < e$.