

Baysian Probability Problems

1. [From *Thinking Fast and Slow* by D. Kahneman, Chapter 16]

A cab was involved in a hit-and run accident at night.

Two cab companies, the Green and the Blue, operate in the city.

Data:

- 85% of the cabs in the city are Green, 15% are Blue.
- A witness identified the cab as Blue. The court tested the reliability of the witness under the circumstances that existed on the night of the accident and concluded that the witness correctly identified each one of the two colors 80% of the time and failed 20% of the time.

What is the probability that the cab that was involved in the accident was Blue rather than Green?

There are two items of information: their *base rate* and the *imperfectly reliable testimony of the witness*. In the absence of a witness, the probability of the guilty can being Blue is 15%, which is the base rate of the outcome.

<http://www.math.upenn.edu/~kazdan/220F12/notes/Bayes/cabs.pdf>

2. Say a 20 year old friend is tested for a relatively rare cancer that occurs in only 1 out of every 1,000 people her age. The test is 90% accurate in the sense that only 10% of those who do not have the cancer still test positive and 98% of those who have the cancer test positive.

- a) If your friend tests positive, what is the likelihood that she has the cancer?
- b) If your friend tests negative, what is the likelihood that she has the cancer?
- c) Repeat parts a) and b) if only 1 out of every 10,000 people her age have the cancer.

<http://www.math.upenn.edu/~kazdan/220F12/notes/Bayes/cancer1.pdf>

3. [This version is only a small modification of the previous problem. Here the test is less effective because only 80% (rather than 98%) of those who have the cancer test positive.]

Say a 20 year old friend is tested for a relatively rare cancer that occurs in only 1 out of every 1,000 people her age. The test is 90% accurate in the sense that only 10% of those who do not have the cancer still test positive and 80% of those who have the cancer test positive.

- a) If your friend tests positive, what is the likelihood that she has the cancer?
- b) If your friend tests negative, what is the likelihood that she has the cancer?
- c) Repeat parts a) and b) if only 1 out of every 10,000 people her age have the cancer.

<http://www.math.upenn.edu/~kazdan/220F12/notes/Bayes/cancer2.pdf>

4. Your next-door neighbor has a rather old and temperamental burglar alarm. If someone breaks into his house, the probability of the alarm sounding is .95. In the last two years, though, the alarm has gone off on five different nights, each time for no apparent reason. Police records show that the chance of a home in your neighborhood being burglarized on any given night are 2 in 10,000. If your neighbor's alarm goes off tonight, what is the likelihood his house is being burglarized?

5. During a power blackout, 100 people are arrested on suspicion of looting. Each is given a polygraph test. From past experience it is known that the polygraph is 90% reliable when administered to a guilty suspect and 98% reliable when given to someone who is innocent. Suppose that of the 100 suspects, only 12 were actually involved in any wrongdoing. What is the probability that a given suspect is innocent given that the polygraph says she is guilty?

<http://www.math.upenn.edu/~kazdan/220F12/notes/Bayes/blackout.pdf>