LAW 520 & MATH 220 & PHIL 220
Problem Set 3
Fall Term, 2012

Due in class on Tuesday, November 27 or in my mail slot in 433 Cohen Hall by 1:00 p.m. the same day. Complete four of the following problems.

1. Let D be the statement $\neg(S(S(\mathbf{0})) = S(S(S(S(\mathbf{0})))))$. Prove D from axioms 1–3 of Memoir 2. Observe that there is a shorter proof using axiom 5.

2. Recall that $\mathbb{N} = \{0, 1, 2, \ldots\}$ is the set of whole numbers. Let f, a function from $\mathbb{N} \times \mathbb{N}$ to $\mathbb{N}$, be defined as follows.

$$f(i, j) = ((i + j + 1)(i + j)/2) + i.$$

   Sketch a proof that f is a bijection, that is, one-one and onto.

3. Use the result of problem 2 to give an explicit algebraic expression $e(i, j, k)$ which establishes a bijection between the set of ordered triples of whole numbers and the set of whole numbers. (You may choose to complete this problem, even if you do not complete problem 2.)

4. Sketch a proof that for every $n \geq 1$, a $(4^n \times 4^n)$-grid with one square removed can be perfectly tiled with L-shaped tiles each of which covers 3 grid squares. (Hint: Use mathematical induction.)

5. Sketch a proof that for every whole number n, the sum $2^0 + 2^1 + 2^2 + \ldots + 2^n$ equals $2^{n+1} - 1$.

6. Let $\mathbf{P}$ be the set of bijections from $\mathbb{N}$ onto $\mathbb{N}$. Sketch a proof that $\mathbf{P}$ is uncountable.

7. Is the square root of a whole number ever rational, but not integral? Sketch a proof for whichever answer you give.

8. Is it the case that for every whole number m, there is a whole number n such that for every whole number k, if $n < k < n + m$, then k is not prime, that is, are there arbitrarily long gaps in the sequence of primes? Sketch a proof for whichever answer you give.