
Signature

PRINTED NAME

Math/Phil 220, Law 540

Exam 1

Oct. 2, 2012

10:30 – 11:30

DIRECTIONS This exam has 5 questions (20 points each). Closed book, no calculators or computers— but you may use one $3'' \times 5''$ card with notes on both sides. *Neatness counts.*

1. Prove that $\sqrt{3}$ is irrational. [If it helps, you may use that if a and b are integers and a prime p divides ab , then it divides either a or b (or both).]
Can you generalize this result?

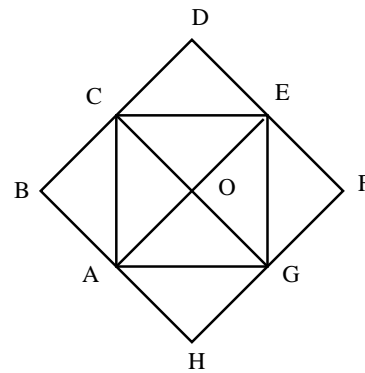
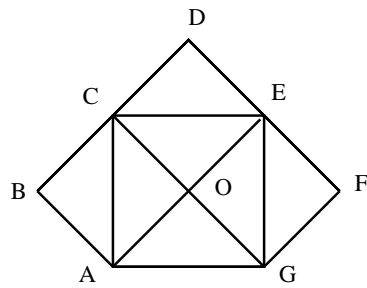
<i>Score</i>	
1	
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2. Prove that the function $f(x) := \sin x + e^x$ is not a polynomial.

3. All odd numbers have the property that when they are divided by 4 their remainder is either 1 or 3. Thus they are either of the form $4k + 1$ or $4k - 1$, where k is an integer. Prove that there are an infinite number of primes of the form $4k - 1$.

[SUGGESTION: Following the idea in Euclid's proof that there are an infinite number of primes, let p be a prime and let $N := 2^2 \cdot 3 \cdot 5 \cdot 7 \cdots p - 1$. Then N has the form $4k - 1$ and is not divisible by any of the primes up to p . Use the observation that the product of any two numbers of the form $4n + 1$ also has this form to show that N cannot be a product of primes only of the form $4n + 1$.]

4. Consider the following two drawings. Show that you can draw the one on the left with a continuous curve – not lifting your pencil from the paper (and not retracing the same line), but you can't do this with the drawing on the right. Explain your reasoning clearly.



[In case it helps, on the last page are additional copies of these figures.]

5. A friend is tested for a relatively rare cancer that occurs in only 1 out of every 10,000 people her age. The test is accurate in the sense that:

- 10% of those who do not have the cancer still test positive (false positives)
- 2% of those who have the cancer test negative (false negatives).

If your friend tests positive, what is the likelihood that she has the cancer?

[There is no need to do any arithmetic to “simplify” your result. I’ll assume you can do that.]

