Proofs in Mathematics, Law and Philosophy: Fall 2012

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Syllabus

Proofs are vital to many parts of life. They arise typically in formal logic, mathematics, the testing of medication, and convincing a jury. How do you prove that the earth is essentially a sphere (in particular, not flat)? In reality, proofs arise anywhere one attempts to convince others. However, the nature of what constitutes a proof varies wildly depending on the situation -- and on whom you are attempting to convince. Convincing your math teacher or a judge is entirely different from convincing your mother or a jury. The course will present diverse views of Proof. On occasion there may be guest lecturers.

The course will devote one month to each of the three segments. Each segment will have homework and a written exam.

Mathematics:

The heart of this course is to achieve some real understanding of proofs in many parts of life. The emphasis will be on mathematical and physical insight and ideas, not complicated formulas. We will begin with some classical examples of proof from number theory: that the square root of 2 is irrational and that there are an infinite number of prime numbers (both were in Euclid's "Elements"). Is .99999... = 1? How do you prove that something exists (the equation $x^5 - 2x - 13 = 0$ has a real root)? How do you prove that something does not exist ("there is no real number whose square is -1")?

Possibly more important than Proofs is asking good questions that lead to the development of a body of knowledge. Two examples we will discuss are the theory of voting and Arrow's Impossibility Theorem" and Bayesian Probability (if you test positive on a cancer test that has false positives, what is the probability that you actually have cancer?)

We will have a guest lecture on how one proves the effectiveness of medication, and, for instance, that smoking and asbestos cause cancer.

The web page http://www.math.upenn.edu/~kazdan/220F12/ will have more information.

Law:

Our goal in the legal section of the course will be to analyze the nature of legal proof: to understand the ways in which mathematical and other techniques of proof are applicable in the law, and how they function in the context of jury trial.

We will discuss many of the differences between a mathematical proof and one in law. One issue we will cover is "confession" as a basic approach to prove guilt --various devices (including torture) to get a confession. Another is that in a criminal case, one needs proof "beyond a reasonable doubt" while in a civil case, only a "preponderance of evidence" is needed; under this rubric we will consider the institutional reasons for "false positives" in the criminal law -- the sources of wrongful convictions, and even of false confessions. Yet a third issue involves the standards of review of videotaped evidence recently enunciated by the Supreme Court in the case of Scott v. Harris, a case which raises the question of how facts are to be proved in legal proceedings, by whom, and under what standards.

Philosophy:

One of our goals in this section of the course is to understand the extent to which philosophical reflection on the nature of mathematical knowledge and mathematical truth has been shaped by the development of mathematical logic through the twentieth century. This will involve careful logical analysis of a number of mathematical proofs and study of elements of the theory of mechanically computable functions.

We will begin our study with a consideration of the second problem posed by Hilbert in his celebrated address to the International Congress of Mathematicians in Paris in 1900. This problem asked for a consistency proof of arithmetic, understood as the theory of the real numbers. In order to appreciate the question itself, we will develop the axiomatic method applied to elementary number theory. We will look at proofs by mathematical induction, expose the elementary theory of divisibility and study the Euclidean algorithm as an example of efficient mechanical computation. We will discuss the concept of logical consequence and will introduce the notions of first-order and second-order logic. We will discuss the Gödel completeness theorem for first-order logic, which indicates the scope for mechanization of mathematical reasoning, and Turing's negative solution to Hilbert's Entscheidungsproblem, which indicates its limitations. We will also discuss Gödel's Incompleteness Theorems, which laid to rest the program Hilbert pursued to resolve the second problem on his 1900 list. We will conclude by discussing a letter Gödel sent to von Neumann in 1956 which was the first intimation of the now famous question concerning the separation of deterministic and nondeterministic polynomial-time Turing computability. In this connection, we will develop elements of the theory of NP-completeness in order to appreciate the significance of the problem. Throughout, we will indicate how various mathematical results have entirely reshaped philosophical thought about the nature of mathematics.