Problem Set 8

DUE: Tues. April 9 in class. [Late papers accepted (without penalty) until 1:00 PM Wed.]

Problems

1. In a certain lake, let $u(t)$ be the number of fish at time t and say it satisfies the modified logistic equation
$$\frac{du}{dt} = u(5-u) - 6,$$
so 6 units are caught in each unit of time.
   a) Find the values of u when $du/dt = 0$ and then, *without solving the equation* describe how the fish population evolves depending on various values of $u(0)$.
   b) Solve the equation explicitly. [Notice that the resulting formula, while of value, do not add much to the qualitative description of part a).]
   c) Repeat part a) describing what changes if only 2 units are caught in each unit of time, so $u'(t) = u(5-u) - 2$.

2. Using the Kermack-McKendrick model, consider an infectious disease with an average infection perion of length $1/a = 7$ days ($R' = aI$) and a per capita rate of contraction of the disease $r = 1/490,000$ per day per individual ($S' = -rSI$). Suppose that the population initially consists of 14 million suseptible people and the number of initial infectives is so small that it can be neglected.
   a) Determine the initial replacement ratio $(r/a)S(0)$.
   b) Determine the maximum number of infectives.
   c) Determine the minimum fraction of the population that needs to be vaccinated in order to prevent an epidemic outbreak.

3. Consider the infectious disease with a and r as above. Assume there are $21,000$ susceptible individuals left after an epidemic. Assume that the number of initial infectives that triggered the epidemic is so small that it can be neglected.

   Determint the maximum number of infectives.

4. Consider a town with $10,000$ susceptible inhabitants. Let the average length of the infectious period be $1/a = 14$ days. Let the infection rate $r = 1/1,000,000$ per day per individual. Give an upper estimate for the final size $S(\infty)$ of the susceptible population.

[Last revised: April 1, 2019]