Math 210, Spring 2019   Jerry L. Kazdan

Problem Set 7

DUE: In class Thurs. Mar. 28 [*Late papers will be accepted until 1:00 PM Fri*].

The first two problems refer to the class notes:
`https://www.math.upenn.edu/~kazdan/210/210F08/LectureNotes/voting/CEU-Voting02.`
`html`
The Internet and other references may be helpful.

Problems

1. This refers to using a Borda count in an election with three candidates. Say there are 10,000 voters each of whom give 3 points to their top choice, 2 points to their second choice, and 1 point to their third choice. The question is how much does this depend on the choice of the numbers 3-2-1? Can the winner change if you use other numbers?

   Clearly if one gives, say, 100 points to the first choise and almost no points to the second and third choices, only the first choice will really matter.

   a) [CHANGE POINTS BY MULTIPLICATION]. Say one gives 30 points to the first choice, 20 to the second, and 10 to the third. Can the outcome of the election change? Either prove that it can't change or give a counterexample showing that it can change.

   To be specific, say one of the candidates is "A". Let $A_{(3,2,1)}$ be the number of A's points using 3-2-1 and $A_{(30,20,10)}$ using 30-20-10. Find a formula for $A_{(30,20,10)}$ in terms of $A_{(3,2,1)}$.

   b) [CHANGE POINTS BY ADDITION]. Say you give 10 points to the first choice, 9 to the second, and 8 to the third, can the outcome of the election change? Proof or counterexample.

   To be specific. find a formula for $A_{(10,9.8)}$ in terms of $A_{(3,2,1)}$.

2. Many people feel that if there is a Condorcet (head-to-head) winner, then that person should win the election.

   Could it happen that someone is a Condorcet winner but would not be a Ranked-Choice "Single Transferable Voting" winner? In this voting procedure a voter ranks all candidates in order of preference. If no one is ranked first by more than 50 percent of voters, the candidate least often ranked first is dropped and their voters' second choices are distributed to the remaining candidates. The process then repeats until a candidate does achieve 50 percent of the top ranking. In that sense, that candidate has majority support and wins.

   Proof or counterexample.

3. [another problem might be added]

[Last revised: May 23, 2019]