Conditional Probability

Notation: $P(A|B)$ means the probability that A is true assuming that B is true.
Two identities:

$$P(A \cap B) = P(A|B)P(B) \qquad P(B \cap A) = P(B|A)P(A)$$

Since $A \cap B = B \cap A$,

$$P(A|B) = \frac{P(B|A)P(A)}{P(B)} \qquad \text{[Bayes]}$$

Dart Board The dart board is the unit disk, $D = \{x^2 + y^2 \le 1\}$. Suppose you know the dart landed in the upper half, $y > 0$. What is the probability thar it landed within a distance of $1/2$ from the origin?

Let $E = \{(x, y) \in D \,:\, y > 0\}$ and $F = \{(x, y) \in D \,:\, x^2+y^2 < (1/2)^2\}$. Then

$$\begin{aligned} P(F|E) =& \frac{P(F \cap E)}{P(E)} = \frac{(1/\pi)(1/2)(\pi/4)}{(1/\pi)(\pi/2)} \\ =& 1/4 \end{aligned}$$

Note that the size of $F \cap E$ is $1/4$ the size of E.

Cancer test. Say you test positive. What is the likelihood you have cancer?

$$P(C|+) = \frac{P(+|C)P(C)}{P(+)} = \frac{P(+|C)P(C)}{P(+|C)P(C) + P(+|-C)P(-C)}$$

Remark:

$P(+|-C)P(-C)$ is the probability of *False Positives*

$P(-|C)P(C)$ is the probability of *False Negatives*

We look at an example with real data.