Signature | PRINTED NAME

Math 210 | # Exam 2 | Jerry L. Kazdan
April 30, 2019 | 1:30–2:50

DIRECTIONS: Part A has 2 shorter questions (10 points each), Part B has 4 traditional problems (20 points each),
To receive full credit your solution should be clear and correct. Neatness counts. You have 1 hour 20 minutes. Closed book, no calculators, but you may use one 3×5 with notes on both sides.

PART A: Two shorter problems, 10 points each [total: 20 points]

Score	
A-1	
A-2	
B-1	
B-2	
B-3	
B-4	
Total	

A-1. Compute the remainder when 3^{1001} is divided by 5.

A-2. The ciphertext message WKHH QGLV QHDU has been encrypted using the Caesar cipher (shift by $+3$). Decrypt it.

In case it helps:

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A B C D E F G H I J K L M N O P Q R S T U V W X Y Z
D E F G H I J K L M N O P Q R S T U V W X Y Z A B C
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PART B: Four traditional problems, 20 points each [80 points]

B-1. Say you seek a parabola with the special form $y = a(x-1)^2 + b$ to pass through the three data points $(0, 2)$, $(1, 0)$, $(2, 3)$.

a) Write the (over-determined) system of equations you would like to solve ideally.

b) Using the method of least squares write the *normal* equations for the coefficients a, b.

c) Explicitly find the coefficients a and b.

B-2. This refers to using a Borda count in an election with three candidates. Say there are 10,000 voters each of whom give 3 points to their top choice, 2 points to their second choice, and 1 point to their third choice. The question is how much does this depend on the choice of the numbers 3-2-1? Can the winner change if you use other numbers?

Clearly if one gives, say, 100 points to the first choise and almost no points to the second and third choices, only the first choice will really matter.

a) [Change points by multiplication]. Say one gives 30 points to the first choice, 20 to the second, and 10 to the third. Can the outcome of the election change? Either prove that it can't change or give a counterexample showing that it can change .

To be specific, say one of the candidates is A. Let $A_{(3,2,1)}$ be the number of A's points using 3-2-1 and $A_{(30,20,10)}$ using 30-20-10. Find a formula for $A_{(30,20,10)}$ in terms of $A_{(3,2,1)}$.

Solution: $A_{(30,20,10)} = 10A_{(3,2,1)}$

b) [Change points by addition]. Say you give 10 points to the first choice, 9 to the second, and 8 to the third, can the outcome of the election change? Proof or counterexample.

To be specific. find a formula for $A_{(10,9.8)}$ in terms of $A_{(3,2,1)}$.

Solution: On each ballot each of the candidates receives 7 more points – regardless of their reanking. Since there are 10,000 voters each candidate receives $70,000$ more points:

$$A_{(10,9,8)} = A_{(3,2,1)} + 7 \times 10,000$$

points

B-3. Using the Kermack-McKendrick model, consider an infectious disease with an average infection perion of length $1/a = 7$ days ($R' = aI$) and a per capita rate of contraction of the disease $r = 1/490,000$ per day per individual ($S' = -rSI$). Suppose that the population initially consists of 14 million suseptible people and the number of initial infectives is so small that it can be neglected.

a) Determine the initial replacement ratio $(r/a)S(0)$.

b) Determine the maximum number of infectives.

c) Determine the minimum fraction of the population that needs to be vaccinated in order to prevent an epidemic outbreak.

B-4. For a simple RSA encryption, you pick $n = pq$, where $p = 3$ and $q = 11$. Say you use the *public exponent* $e = 3$.

a). Find the *private exponent* d.

b). Say the entire message Alice wants to send you is the number 6. What is Alice's encryption of this message?

c). Breifly *describe* the computation you would need to do to decrypt Alice's message. (You are not asked to complete the calculation).

[Blank page for scratch work]