a) If her balance is to decrease, then her first monthly payment must be more than the interest accumulated in the first month. If she does this, then her balance at the second month will be less than her first month and her accumulated interest will fall even more. This cycle would continue until she has paid off her entire debt. Her interest accumulated after the first month is $\frac{p_0\, i}{12}$. Therefore, her monthly payment M must be more than $\frac{p_0\, i}{12}$.

b) Let us take the suggestion and substitute c for $1+\frac{i}{12}$. We know that $p_1 = c\,p_0 - M$. Therefore, $p_2 = c\,p_1 - M$. Substituting $c\,p_0 - M$ for p_1 in the second equation, we get that $p_2 = c\,(c\,p_0 - M) - M$. This simplifies to $p_2 = c^2\,p_0 - M\,(1+c)$.

c) Continuing the process from part (b), we can find p_3:

$$p_3 = c\,p_2 - M$$

$$p_3 = c\,(c^2\,p_0 - M\,(1+c)) - M$$

$$p_3 = c^3\,p_0 - \mathrm{M}(1+c+c^2)$$

p_4 and p_5 and p_k can be found in the same way. The general formula should be apparent:

$$p_4 = c^4\,p_0 - M\,(1+c+c^2+c^3)$$

$$p_5 = c^5\,p_0 - M\,(1+c+c^2+c^3+c^4)$$

$$p_k = c^k\,p_0 - M\,(1+c+c^2+\text{"..."}+c^{(k-2)}+c^{(k-1)})$$

The sum $1+c+c^2+\text{"..."}+c^{(k-1)}$ is a geometric summation. The summation is equivalent to $\frac{1-c^k}{1-c}$.

$$p_k = c^k\,p_0 - \frac{M\,(1-c^k)}{1-c}.$$

Substituting $1+\frac{i}{12}$ for c, we get:

$$p_n = \left(1+\frac{i}{12}\right)^k p_0 - \frac{M\left(1-\left(1+\frac{i}{12}\right)^k\right)}{1-\left(1+\frac{i}{12}\right)}.$$

We can simplify the bottom, since $1-\left(i+\frac{i}{12}\right)$ is equal to $-\frac{i}{12}$:

$$p_k = \left(1+\frac{i}{12}\right)^k p_0 + \frac{12\,M\left(1-\left(1+\frac{i}{12}\right)^k\right)}{i}$$

d) To solve for N, the total number of monthly payments, we want to know when $p_k \le 0$. In our equation, the constants are i and p_0, while our variables are n and M. Therefore, if we want to find N, given M, we should solve for k. We can check our answer by plugging in values for M, i, and p_0:

```
> solve(0 = (1+i/12)^k*p[0]+12*M*(1-(1+i/12)^k)/i,k);
> N = evalf(subs({M=350, i=0.08, p[0]=10000}, %));
```

$$\frac{\ln\left(-12\,\frac{M}{p_0\,i-12\,M}\right)}{\ln\left(1+\frac{1}{12}\,i\right)}$$

$$N = 31.80190006$$

e) To solve for M, we simply do a similar process as above. Again, we want to know when $p_k \leq 0$. However, since we want to find M, given k, we should solve for M:

```
> solve(0 = (1+i/12)^k*p[0]+12*M*(1-(1+i/12)^k)/i,M);
> M = evalf(subs({k=24, i=0.08, p[0]=10000}, %));
```

$$\frac{1}{12}\,\frac{\left(1+\frac{1}{12}\,i\right)^k p_0\,i}{-1+\left(1+\frac{1}{12}\,i\right)^k}$$

$$M = 452.2728939$$