```
> restart;
```

Here's the equation f; let's find the derivative, set it to zero, and see the best k in terms of p.

```
> f := ((1-p)^k + (1 - (1-p)^k)*(k+1))*(n/k);
```

$$f := \frac{((1-p)^k + (1-(1-p)^k)(k+1))\,n}{k}$$

```
> df := diff(f,k);
```

$$df := \frac{((1-p)^k \ln(1-p) - (1-p)^k \ln(1-p)(k+1) + 1 - (1-p)^k)\,n}{k} - \frac{((1-p)^k + (1-(1-p)^k)(k+1))\,n}{k^2}$$

```
> k = solve(df=0,k);
```

$$k = \left(2\,\frac{\mathrm{LambertW}\left(\frac{1}{2}\sqrt{-\ln(1-p)}\right)}{\ln(1-p)},\ 2\,\frac{\mathrm{LambertW}\left(-\frac{1}{2}\sqrt{-\ln(1-p)}\right)}{\ln(1-p)} \right)$$

This is the <u>exact</u> answer to what k should be. If anyone in the class knows how to use the Lambert W equation, he/she shouldn't be in this class. Instead, we need to find a simplification to this problem. Through the binomial theorem we know that (1-n)^r for small enough n is approximately (1-nr). We can use this in our equation.

```
> g := ((1 - p*k) + (1 - (1 - p*k))*(k+1))*n/k;
```

$$g := \frac{(1 - p\,k + p\,k\,(k+1))\,n}{k}$$

```
> g := simplify(g);
```

$$g := \frac{(1 + p\,k^2)\,n}{k}$$

```
> dg := diff(g,k);
```

$$dg := 2\,p\,n - \frac{(1+p\,k^2)\,n}{k^2}$$

This looks like a more reasonable answer. We'll take the positive root in this case, which is 1 / sqrt(p).

```
> k = solve(dg=0,k);
```

$$k = \left(\frac{1}{\sqrt{p}}, -\frac{1}{\sqrt{p}} \right)$$

Let's check our answers by plugging in values for p, n, and k. Notice how functions f and g give us almost exactly the same answer (which is the expected number of tests performed at this drug percentage, total number of people, and size of group.

```
> p:=0.01; n:=10000; k:=1/sqrt(p);
```

$$p := .01$$

$$n := 10000$$

$$k := 10.00000000$$

```
> evalf(f);
```

$$1956.179250$$

```
> evalf(g);
```

$$2000.000000$$

```
>
```