**[Who Solved the Secretary Problem?]: Comment**

Peter R. Freeman

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# Comment

**Peter R. Freeman**

Professor Ferguson has written an entertaining paper that for me had the unputdownable qualities of a good detective novel as I followed all the twists and turns of the plot on the way to discovering who solved it. There can't be many other stories, surely, in which the author takes over and performs the deed himself on the final page.

I must begin by taking the opportunity of recording my personal debt to the two papers by Lindley and by Gilbert and Mosteller. It was their elegance and beautiful clarity that first kindled by own interest in the secretary problem and in other sequential decision problems, especially those relating to statistical inference. They still vividly convey the excitement and sheer fun of proposing and solving a whole series of increasingly complex problems, giving new graduate students a better idea of what it's like to do successful research in applied probability than anything else I know.

The secretary problem also serves as an excellent case-study of the evolution of research. It starts as an intriguing, not exactly practical but at least realistic, problem. It quickly leads into a branching process at each node of which one of the simple assumptions can be replaced by something more general. Before we know where we are (well, within 25 years, anyway), one problem has turned into hundreds and an army of academics is solving them while simultaneously inventing thousands more. This is all understandable and to some extent admirable, but as one whose interests have turned ever more towards practicalities over the years, I have to ask the embarrassing question: Has anyone *ever* used a secretary-type approach to solve *any* real problem of practical importance? Is anybody willing to admit, for example, to having actually chosen a secretary or a wife using the "optimal" policy?

More seriously, the most common defect in papers that I get to referee in this area is that they assume rather than prove that the optimal policy will be of the form "reject the first $r - 1$ applicants, then accept the first that . . .". This is, of course, not necessarily true, as Presman and Sonin (1972) were the first to show. I'd be very interested to know what is currently the strongest statement that can be made about conditions under which it *is* true. I suspect that there is still fame, if not fortune, awaiting the first person to make progress with the deep issues underlying sequential optimality.

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# Rejoinder

**Thomas S. Ferguson**

One aspect brought out by the comments of the discussants of this paper is the vitality and robustness of the class of problems engendered by the secretary problem. Some new results and some open problems are mentioned in passing. There's life in the old secretary yet!

So much so that my title has come in for criticism. Steve Samuels has punctured my balloon by pointing out that there is still a point to be resolved in what I call *the* secretary problem. According to him, it is not enough to show that the problem can be solved to within $\varepsilon$ for every $\varepsilon > 0$; one must also determine whether or not it can be solved for $\varepsilon = 0$. I admit that as I have worked on this problem (since 1961) I was under the impression that no optimal strategy existed for the player who chooses the set of numbers, and that elementary methods should suffice to show this. Now that he points it out, I can see that the proof of this conjecture, if indeed it is true, is by no means easy, requiring, as it does, a strengthening of Hill's result. Yet, I would be very surprised if one could find an optimal strategy for the numbers chooser, and so I am willing to conjecture that no such strategy exists. I admit I am on shaky ground as I can see no reason for the validity of Samuels' condition (a) to imply the invalidity of his condition (b)—and this is just for the case $n = 3$.

This mathematically interesting open problem prompts me to ask if the secretary problem will ever be "solved." Maybe it's like the central limit