```latex
\documentclass[11pt]{article}
%Options: draft shows overfull lines, reqno-leqno puts eq numbers on right/left
%\documentclass[11pt,draft, reqno,a4paper,psamsfonts]{amsart}

\usepackage{amsmath}
\usepackage{amssymb}
\usepackage{graphicx}           % to include graphics

%           *** CHANGE DIMENSIONS ***
\voffset=-0.3truein     % LaTeX has too much space at page top
\addtolength{\textheight}{0.3truein}
\addtolength{\textheight}{\topmargin}
\addtolength{\topmargin}{-\topmargin}
\textwidth  6.0in       % LaTeX article default 360pt=4.98''
%\parindent=20pt

%           *** MACROS ***
%MATH Macros
\newcommand{\R}{\mathbb{R}}  %blackboard bold R
\newcommand{\C}{\mathbb{C}}
\newcommand{\F}{\mathbb{F}}
\newcommand{\N}{\mathbb{N}}
\newcommand{\Z}{\mathbb{Z}}
\newcommand{\abs}[1]{\lvert #1 \rvert}      % absolute value
\newcommand{\norm}[1]{\lVert #1 \rVert}     % norm
\newcommand{\ip}[2]{\langle #1,\, #2\rangle}    % ip = inner product

%==================== END PREAMBLE ========================================
\begin{document}
% Begin body of article here.
%\pagestyle{empty}
\parindent=0pt
\vspace*{-1.0cm}

{\large Math 202 \hfill Jerry L. Kazdan}

\medskip
\begin{center}
\fbox{\large $\displaystyle\mathbf \sin x + \sin 2x+\cdots+\sin nx =
\frac{\cos\frac{x}{2} - \cos(n+\frac{1}{2})x}{2\sin\frac{x}{2}} $}
\end{center}

\bigskip
The key to obtaining this formula is either to use some imaginative
trigonometric identities or else recall that $e^{ix}=\cos x + i\sin x$
and then routinely sum a geometric series.  I prefer the later.  Thus
\begin{equation} \label{sum1}
 \sin x + \sin 2x+\cdots+\sin nx
 =\text{Im}\{e^{ix} + e^{i2x} + \cdots + e^{inx}\},
 \end{equation}
where \,Im$\{z\}$ means take the imaginary part of the complex number $z=x+iy$.
The sum on the right side is a (finite) geometric series $t+t^2+\cdots t^n$
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```latex
where $t=e^{ix}$:
\begin{equation*}
t+t^2+\cdots+t^n=\frac{t(1 - t^{n})}{1-t}.
\end{equation*}
Thus
\begin{equation}\label{sum2}
\sin x + \sin 2x+\cdots+\sin nx
 =\text{Im}\bigg\{\frac{e^{ix}(1 - e^{inx})}{1-e^{ix}}\bigg\}.
 \end{equation}

We need to find the imaginary part of the fraction on the right.
The denominator is what needs work.  By adding and subtracting
\[
 e^{i\theta} = \cos\theta + i\sin\theta \quad\text{and}\quad
e^{-i\theta} = \cos\theta - i\sin\theta
\]
we obtain the important formulas
\[
\cos\theta = \frac{e^{i\theta}+e^{-i\theta}}{2} \quad \text{and}\quad
\sin\theta = \frac{e^{i\theta}-e^{-i\theta}}{2i}.
\]
Thus
\[
1-e^{ix}=e^{ix/2}\left(e^{-ix/2} - e^{ix/2}\right) =-2ie^{ix/2}\sin\frac{x}{2}
\]
so
\begin{equation}\label{sum3}
\frac{e^{ix}(1 - e^{inx})}{1-e^{ix}}
= i\left[\frac{e^{i\frac{x}{2}} - e^{i(n+\frac{1}{2})x}}{2\sin{\frac{x}{2}}}
\right].
\end{equation}
Consequently, from \eqref{sum3}, taking the imaginary part of the
right side (so the real part of $[\cdots]$) we obtain the desired formula:
\[
\sin x + \sin 2x+\cdots+\sin nx =
\frac{\cos\frac{1}{2}x - \cos(n+\frac{1}{2})x}{2\sin\frac{1}{2}x}
\]
\medskip
{\sc Remark:} By taking the real part in \eqref{sum3} we obtain the
related formula
\[
\cos x + \cos 2x +\cdots+\cos nx
=\frac{-\sin\frac{1}{2}x + \sin(n+\frac{1}{2})x}{2\sin\frac{1}{2}x}.
\]

{\sc Exercise:} Use \ $\sin(a+x)+\sin(a+2x)+\cdots+\sin(a+nx)
=\text{Im}\{e^{ia}(e^{ix}+\cdots+e^{inx})\}$ to compute a formula for \
$\sin(a+x)+\sin(a+2x)+\cdots+\sin(a+nx)$.  [Taking the derivative of
this formula with respect to $a$ gives another route to the formula of
the Remark just above.]

\bigskip
```

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[Last revised: \today]
\end{document}
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