Big Numbers

- Age of the universe (14 billion years) in seconds:

$$\begin{aligned} N =& 14 * 10^9 \text{ years } * \frac{365 \text{ days}}{\text{year}} * \frac{24 \text{ hours}}{\text{day}} * \frac{3600 \text{ seconds}}{\text{hour}} \\ =& 441,504,000 * 10^9 \approx 4.4 * 10^{17} \text{ seconds.} \end{aligned}$$

- 1 billion $= 1,000,000,000 = 10^9$
- $2^{121} = (2^{11})^{11} = 2048^{11} \approx 2.7\ 10^{36}$
- $9^{55} = (9^5)^{11} = 32805^{11} \approx 3.0\ 10^{52}$
- $7^{88} = (7^8)^{11} = 5764801^{11} \approx 2.3\ 10^{74}$
- $e^{100} \approx 2.68\ 10^{43}$
- $15! \approx 1.3\ 10^{12} \qquad 25! \approx 1.5\ 10^{25}$

Example 1. Let $S_N := 1 + \frac{1}{2} + \cdots + \frac{1}{N}$. Find an estimate for N so that $S_N > 100$.

ANSWER[1] By the idea behind the *integral test*

$$\ln(N+1) < S_N < 1 + \ln N$$

Thus, to insure that $S_N > 100$ pick $\ln(N+1) > 100$, that is, $N + 1 > e^{100} \approx 2.7 * 10^{43}$.

On the other hand, if $1 + \ln N < 100$, then $S_N < 100$. Here we can pick any $N < e^{99} \approx 10^{43}$.

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[1] R. P. Boas, Jr. and J. W. Wrench, Jr. found the exact number:

15092688622113788323693563264538101449859497

Amer. Math. Montly, Vol. 78, No. 8, Oct., 1971, pp 864–870, DOI: 10.2307/2316476, Stable URL: https://www.jstor.org/stable/2316476

Example 2. How many multiplications are needed to compute the determinant of a 25×25 matrix of real numbers?

ANSWER: $25! \approx 10^{25}$

Example 3. Consider a stock that is sold on the New York Stock Exchange. If someone is Paris places a buy order and 1/100 of a second later someone in New York buys the same stock, which order is executed first?

Some Data:

$$\begin{aligned}\text{Speed of light in a vacuum} &\approx 300,000 \text{ km/sec} \\ &\approx 186,000 \text{ miles/sec} \\ &\approx 1 \text{ foot/ns} \approx 1 \text{ m}/3.3\,\text{ns} \quad (\text{ns} = \text{nanoseconds}) \\ &\approx 4,000 \text{ miles}/.0215 \text{ sec} \quad (\text{3,625 miles from Paris to New York}\end{aligned}$$

Significant Digits

Say we have the data $a := 12.47$ and $b := 7.2$, both rounded off. Thus the absolute value of the error in a is at most $e_a : 0.005$ while $e_b := 0.05$. Now $ab = 89.784$. How trustworthy is this? Since

$$12.465 < a < 12.475 \qquad \text{while} \qquad 7.15 < b < 7.25,$$

we know that

$$89.12475 < ab < 90.44375.$$

We should discard most of the digits and use the rounded tumber

$$ab \approx 89.8.$$

To better understand the error, note that

$$(a + e_a)(b + e_b) = ab + ae_b + be_a + e_a e_b \approx ab + ae_b + be_a.$$

The larger number, a, happens to b multiplied by the larger error, e_b. Although a has 4 significant digits, b only has 2. The analysis changes considerably if you are given that $b = 7.20$ since then $e_b = 0.005$.

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