

Problem Set 5

DUE: In class Thursday, Oct. 11. *Late papers will be accepted until 1:00 PM Friday.*

REMARK: Please read Chapter 15 pages 293 – 304 on Continuity.

PROBLEMS

1. [#14.32] A runaway train is hurtling toward a brick wall at a speed of 100 miles per hour. When it is 2 miles from the wall, a (speedy) fly begins to fly repeatedly between the train and the wall at the speed of 200 miles per hour. Determine how far the fly travels before it is smashed.

2. [#14.36] Find the rational number whose repeating decimal expansion is $.247247247\dots$

3. Find all points in the complex plane where $\sum_0^{\infty} \frac{(z - 3i)^n}{n^2 2^n}$ converges.

4. Find all points in the complex plane where $\sum_0^{\infty} \frac{1}{(z - 3i)^n}$ converges.

5. Show that the power series $\sum_{n=0}^{\infty} (-1)^n \frac{z^{2n}}{(2n)!}$ converges for all $z \in \mathbb{C}$.

[REMARK: In calculus, for real $z = x$ this was found to be the Taylor series for $\cos x$. For complex z , this convergent series is used to *define* $\cos z$].

6. In class – and in the book – we showed that the *harmonic series* diverges to infinity.

a) Find an integer N so that

$$1 + \frac{1}{2} + \frac{1}{3} + \cdots + \frac{1}{N} > 100.$$

b) If you calculate this sum of N terms on a computer, say computing 1,000 terms per second, how many years will it take?

7. [#15.4] There is an $x \in \mathbb{R}$ so that $\frac{x^2 + 5}{3 + x^7} = 1$. Why or why not?

8. [15.8] let $f : \mathbb{R} \rightarrow \mathbb{R}$. If $|f|$ is continuous, then f is continuous. Proof or counterexample.

Bonus Problems

[Please give your solutions directly to Professor Kazdan]

1-B COMPLEX ROOTS OF POLYNOMIALS. Let $p(z) = z^n + a_{n-1}z^{n-1} + a_{n-2}z^{n-2} + \cdots + a_1z + a_0$ be a polynomial of degree n . Find a real number R (depending on the coefficients a_j) such that all of the (possibly complex) zeroes of p are in the disk $\{|z| \leq R\}$ in the complex plane.

REMARK: There are many elementary approaches ("elementary" does not mean obvious). It may simplify your approach if you find this estimate only for the roots outside the unit disk, so $|z| > 1$.

[Last revised: October 8, 2018]