

Problem Set 10

DUE: In class Thursday, Nov. 29 *Late papers will be accepted until 1:00 PM Friday.*

PROBLEMS

- [# 17.6] Let f and g be bounded real-valued functions on a set S .
 - Prove that $\sup_S(f + g) \leq \sup_S f + \sup_S g$.
 - Give an example where strict inequality holds.
- [#17.7] Let $f(x) = x^2$, and let P_n be a partition of the interval $[0, 3]$ into n intervals of equal length.
 - Compute formulas for $L(f, P_n$ and $U(f, P_n)$ in terms of n . Verify that they have the same limit.
 - Find a number n to insure that $U(f, P_n)$ is within .01 of $\int_0^3 x^2 dx$.
- Let $f(x) = 0$ for all $x \in [0, 2]$ *except* at $x = 1$ where $f(1) = 3$. Show that f is Riemann integrable on $[0, 2]$ – and
 Compute the value of the integral.

- Let $f(x) = \sin(1/x)$ for $x \neq 0$ and $f(0) = -3$. Show that f is Riemann integrable on the interval $[-1, 1]$.

- [#17.13] Let $f(x)$ be continuous for $x \in [a, b]$.
 - Show there is some point $c \in [a, b]$ where f has its average value, that is,

$$f(c) = \frac{1}{b-a} \int_a^b f(x) dx$$

[SUGGESTION: First do the case where $\int_a^b f(x) dx = 0$.

- If f is not continuous, there may not be any such point c . Give an example.
- If $\int_0^x f(t) dt = x \cos(\sin x) + C$, find the continuous function f and the constant C .
 - [#17.16] For $x > 0$ let $g(x) := \int_0^x \frac{1}{1+t^2} dt + \int_0^{1/x} \frac{1}{1+t^2} dt$. Show that g is a constant.

8. For which powers $p > 0$ does the series $\sum_{k=2}^{\infty} \frac{1}{k(\ln k)^p}$ converge? [HINT: integral test.]

9. Compute $\lim_{\lambda \rightarrow \infty} \int_0^1 |\sin(\lambda x)| dx$.

10. Let $f(t)$ be a continuous function for $0 \leq t < \infty$. If $\lim_{t \rightarrow \infty} f(t) = c$, show that

$$\lim_{T \rightarrow \infty} \frac{1}{T} \int_0^T f(t) dt = c.$$

[HINT: This is like Proposition 14.11 on page 275 in the text.]

Bonus Problems

[Please give your solutions directly to Professor Kazdan]

1-B Let f be continuous on the interval $[0, \pi]$. Show that $\lim_{\lambda \rightarrow \infty} \int_0^{\pi} f(x) \sin(\lambda x) dx = 0$.

[Last revised: November 26, 2018]