Math 202 — October 2, 2018

Exam 1

Jerry L. Kazdan — 10:30 — 11:50

Directions: Part A has 8 shorter problems (5 points each) while Part B has 4 traditional problems (10 points each). To receive full credit your solution should be clear and correct. Neatness counts. You have 1 hour 20 minutes. Closed book, no calculators, but you may use one 3 $\times 5$ with notes on both sides.

Part A: Eight shorter problems, 5 points each.

A-1. If $a_1 = 1$ and $a_{n+1} = \sqrt{3a_n + 4}$, show that $a_n < 4$ for all $n = 1, 2, 3, \ldots$.

A-2. Show that $\sqrt{3}$ is not a rational number.

A-3. Show that $\lim_{n\to\infty} \frac{5^n}{n!} = 0$

A-4. Give an example of a sequence of real numbers that is not monotone but that converges to some limit.

A-5. Give an example of a sequence x_n of real numbers with at least two subsequences that converge to different limits.

A-6. Give an example of an *unbounded* sequence of real numbers a_n that satisfies $|a_{n+1} - a_n| \to 0$.

A-7. Determine if the series $1 + \frac{1}{3} + \frac{1}{5} + \cdots + \frac{1}{2n+1} + \cdots$ converges or diverges. Explain your reasoning.

A-8. [Proof or Counterexample] Let a_n be a sequence of real numbers that converges to L. Then there exists an $\epsilon > 0$ such that for all integers n we have $|a_{n+1} - a_n| < \epsilon$.

Part B: Four traditional problems, 10 points each.

B-1. Let a_n and b_n be sequences of of complex numbers. If $a_n \to A$ and $b_n \to B$, show that $a_n b_n \to AB$. [Give a formal proof using ϵ and N.]

B-2. Let $a_n = \sqrt{n^2 + 6n} - n$. Show that a_n converges and find the limit.

B-3. Let a_n be a sequence of real numbers that converge to L. If $L > 0$, show there is an N so that if $n > N$ then $a_n > \frac{1}{2}L$.

B-4. Let $a_n \to A$ and $b_n \to B$ be convergent sequences of real numbers, and let c_n be the larger of a_n and b_n, so $c_n = \max(a_n, b_n)$. Either prove that this sequence c_n converges or give a counterexample.

Remark: There are three cases: $A < B$, $A > B$, and $A = B$.