Math 202, Fall 2013 Jerry L. Kazdan

Problem Set 11

DUE: In class Thursday, Dec. 5 *Late papers will be accepted until 1:00 PM Friday.*

REMARKS: Please read Chapter 17 on the integral.

PROBLEMS

1. [# 17.6] Let f and g be bounded real-valued functions on a set S.
   a) Prove that $\sup_S(f+g) \le \sup_S f + \sup_S g$.
   b) Give an example where strict inequality holds.

2. [#17.7] Let $f(x) = x^2$, and let P_n be a partition of the interval $[0, 3]$ into n intervals of equal length.
   a) Compute formulas for $L(f, P_n$ and $U(f, P_n)$ in terms of n. Verify that they have the same limit.
   b) Find a number n to insure that $U(f, P_n)$ is within .01 of $\int_0^3 x^2\,dx$.

3. Let $f(x) = 0$ for all $x \in [0, 2]$ *except* at $x = 1$ where $f(1) = 3$. Show that f is Riemann integrable on $[0, 2]$.

4. [#17.13] Let $f(x)$ be continuous for $x \in [a, b]$.
   a) Show there is some point $c \in [a, b]$ where f has its average value, that is,
   $$f(c) = \frac{1}{b-a}\int_a^b f(x)\,dx$$
   [SUGGESTION: First do the case where $\int_a^b f(x)\,dx = 0$. Then reduce the general case to the special case by using $g(x) := f(x) - \frac{1}{b-a}\int_a^b f(t)\,dt$.]
   b) If f is not continuous, there may not be any such point c. Give an example.

5. If $\displaystyle\int_0^x f(t)\,dt = x\cos(\sin x) + C$, find the continuous function f and the constant C.

6. [#17.16] For $x > 0$ let $\displaystyle g(x) := \int_0^x \frac{1}{1+t^2}\,dt + \int_0^{1/x} \frac{1}{1+t^2}\,dt$. Show that g is a constant.

7. For which powers $p > 0$ does the series $\displaystyle\sum_{k=2}^{\infty} \frac{1}{k(\ln k)^p}$ converge? [HINT: integral test.]

8. [#17.26] Use Theorem 17.26 to find the indefinite integrals of $\ln x$ and $\tan^{-1} x$. [Here $\tan^{-1} x$ is the "arc tangent of x"].

9. Compute $\lim_{\lambda\to\infty} \int_0^1 |\sin(\lambda x|\, dx$.

10. Let $f(t)$ be a continuous function for $0 \le t < \infty$. If $\lim_{t\to\infty} f(t) = c$, show that

$$\lim_{T\to\infty} \frac{1}{T} \int_0^T f(t)\, dt = c.$$

[HINT: This is like Proposition 14.11 on page 275 in the text.]

Bonus Problems

[Please give your solutions directly to Professor Kazdan]

1-B Let f be continuous on the interval $[0, \pi]$. Show that $\lim_{\lambda\to\infty} \int_0^\pi f(x)\sin(\lambda x)\, dx = 0$.

2-B Let $f(x) = \sin(1/x)$ for $x \neq 0$ and $f(0) = 2$. Show that f is Riemann integrable on the interval $[0, 1]$.

[Last revised: December 22, 2013]