Problem Set 1

DUE: In class Thursday, Sept. 5. *Late papers will be accepted until 1:00 PM Friday.*

I enjoyed meeting you yesterday, and look forward to our semester together.

In our textbook, *Mathematical Thinking, Problem-Solving and Proofs*, Second edition, by John P. D'Angelo and Douglas B. West, read Chapter 1, pages 2 - 20. If something is not clear, give it a little attention, but then move on rather than allowing yourself to get stuck. Do this part BEFORE next week's class.

1. [Text, p.20 #1.4] Explain why the square has the largest area among all rectangles with a given perimeter.

2. [Text, p.21 #1.8] In the morning section of a calculus course, 2 of the 9 women and 2 of the 10 men receive the grade of A. In the afternoon section, 6 of the 9 women and 9 of the 14 men receive the grade of A. Verify that in each section a higher proportion of women than of men receive A, but that in the combined course a lower proportion of women than of men receive A. Explain!

3. [Text, p.21 #1.19] What are the dimensions of a rectangular carpet with perimeter 48 feet and area 108 square feet? Given positive numbers p and a, under what conditions does there exist a rectangular carpet with perimeter p and area a?

4. [Text, p.22 #1.22] We have two identical glasses. Glass 1 contains x ounces of wine; glass 2 contains x ounces of water ($x \geq 1$). We remove 1 ounce of wine from glass 1 and add it to glass 2. The wine and the water in glass 2 mix uniformly. We now remove one ounce of liquid from glass 2 and add it to glass 1. Prove that the amount of water in glass 1 is the same as the amount of wine in glass 2.

5. [Text, p.22 #1.24] Three people register for a hotel room; the desk clerk charges them a total of $30. The manager returns and says this was an overcharge, instructing the clerk to return $5. the clerk takes five $1 bills. but pockets $2 as a tip and returns only $1 to each guest. Of the original $30 payment, each guest actually paid $9 and $2 went to the attendant. What happened to the "missing" dollar?

6. [Text, p.22 #1.25] A census taker interviews a woman in a house. "Who lives here?" he asks. "My husband and I and my three daughters," she replies. "What are the ages of your daughters?" "The product of their ages is 36 and the sum of their ages is the house number."

   The census taker looks at the house number, thinks, and says, "You haven't given me enough information to determine the ages." Oh, you're right," she replies. "Let me

also say that my eldest daughter is asleep upstairs." "Ah! Thank you very much." What are the ages of the daughters? [The problem requires "reasonable" mathematical interpretation of its words.]

7. Prove that $\sqrt{7}$ is not a rational number.

8. A polynomial is a function of the form $p(x) = a_0 + a_1 x + \cdots + a_n x^n$ where the coefficients $a_0, \ldots, a_h$ are constants. If $a_n \neq 0$, then the *degree* of this polynomial is n.

   Show that the exponential function e^x is not a polynomial.

9. In the text on page 140 read the definition of an *equivalence relation*. [For the set S of all students in Math 202, an example is two students are "equivalent" if they have the same birthday (perhaps in different years).]

   For the set $\mathbb{Z}$ of integers, given a positive integer n, we say that two integers x and y to be *congruent* mod n *if* $x - y$ *is divisible by* n. *We write this as* $x \equiv y \mod n$. [EXAMPLE: $-2 \equiv 7 \mod 3$]. Clocks show the time of day mod 12.

   a) Show that $x \equiv y \mod n$ is an equivalence relation.

   b) If $a \equiv b \mod n$ and $c \equiv d \mod n$, show that both of the following are true:

   $$ a + c \equiv b + d \mod n \qquad \text{and} \qquad ac \equiv bd \mod n $$

   c) As an application, use $10 \equiv 1 \mod 9$ to show that $10^2 \equiv 1 \mod 9$. In fact for any positive integer k, then $10^k \equiv 1 \mod 9$.

[Last revised: August 4, 2018]