Homework Set 7 (Due in class on Thursday, Nov. 5)

(late papers accepted until 1:00 Friday)

The problem numbers refer to the D'Angelo-West text.

1. [#14.50] Determine if $1+\frac{1}{3}+\frac{1}{5}+\frac{1}{7}+\frac{1}{9}+\cdots$ converges.

2. In class we proved that the harmonic series $\sum \frac{1}{n}$ diverges. Use the estimate in our proof to find an integer N so that
$$1+\frac{1}{2}+\frac{1}{3}+\frac{1}{4}+\cdots+\frac{1}{N}>100.$$

3. [#14.59] Let a_n be a convergent sequence of positive real numbers. Prove that $\sum_1^\infty \frac{1}{na_n}$ diverges.

4. [#14.60] Determine if each of the series below converges or diverges.
$$(a)\quad \sum_1^\infty \frac{2n^2+15n+2}{n^4+3n+1} \qquad (b)\quad \sum_1^\infty \frac{2n^2+15n+2}{n^3+3n+1} \qquad (c)\quad \sum_1^\infty \frac{3+5n+n^2}{2^n}$$

5. Find the disk of convergence (in the complex plane) of the following power series:
$$(a)\quad \sum_0^\infty \frac{n^2(z-2)^{2n}}{3^n} \qquad (b)\quad \sum_0^\infty \frac{n^2(z-2i)^n}{3n!} \qquad (c)\quad \sum_0^\infty n!(z+3)^n$$
Note: only find the center and radius of this disk. You are not being asked to determine the convergence on the boundary of the disk of convergence.

6. [#14.52] Alternating Series Test Let c_k be a sequence of positive real numbers that is monotone decreasing to zero. Show that the alternating series
$$c_1-c_2+c_3-c_4+c_5-c_6+\cdots$$
converges. [As a model, look at the proof we did in class that $1-\frac{1}{2}+\frac{1}{3}-\frac{1}{4}+\frac{1}{5}-\cdots$ converges.]

7. [e is irrational]. We defined the number e by the power series $e=\sum_0^\infty \frac{1}{n!}$. The point of this problem is to show that e is irrational.

a) Show that $2 < e < 3$. So e is definitely not an integer.

b) By contradiction, say $e = \frac{p}{q}$, where p and q are positive integers with $q \geq 2$. Show that

$$e\,q! = N + \frac{c}{q+1},$$

where N is an integer and $0 < c < e$. Thus, conclude that $\frac{c}{q+1}$ must be an integer.

c) Then show that this contradicts $e < 3$ and $q + 1 \geq 3$.

[Last revised: November 6, 2009]