

## Homework Set 6 (Due in class on Thursday, Oct. 29)

(late papers accepted until 1:00 Friday)

The problem numbers refer to the D'Angelo-West text.

1. [#14.24] Let  $f(x) := x^2 - 4x + 6$  and let  $x_n$  be a sequence defined by the recurrence  $x_{n+1} = f(x_n)$ , that is,  $x_{n+1} = x_n^2 - 4x_n + 6$ .
  - a) If  $\lim_{n \rightarrow \infty} x_n$  exists and equals  $L$ , what possible values can  $L$  have?
  - b) The behavior as  $n \rightarrow \infty$  depends on the initial value,  $x_0$ . For each  $x_0 \in \mathbb{R}$ , describe this behavior. [HINT: Graph the functions  $y = x$  and  $y = f(x)$  and interpret the graphs. In this example, it may help to rewrite  $y = f(x)$  as  $y - 2 = (x - 2)^2$ .]
2. [#14.27] For  $c > 0$ , let  $x_n = (c^n + 1)^{1/n}$ . Determine  $\lim_{n \rightarrow \infty} x_n$ . More generally find  $\lim_{n \rightarrow \infty} (a^n + b^n)^{1/n}$ . [HINT: First consider the case  $c < 1$  and use the Squeeze Theorem.]
3. [#14.30] Let  $x_n$  be the sequence defined recursively by  $x_1 = 1$  and  $x_{n+1} = 1/(x_1 + \cdots + x_n)$ . Prove that this sequence converges and obtain the limit.
4. [#14.33] If  $\sum_{k=1}^{\infty} a_k$  converges to  $A$  and  $\sum_{k=1}^{\infty} b_k$  converges to  $B$ , show that  $\sum_{k=1}^{\infty} (a_k + b_k)$  converges to  $A + B$ .
5. [#14.36] Find the rational number whose repeating decimal expansion is  $.247247247\dots$ . Also, find the rational number whose repeating octal (that is, base 8) expansion is  $.247247247\dots$ .
6. [#14.43] Compute  $\sum_{k=1}^{\infty} \left(\frac{x}{x+1}\right)^k$ . What assumptions must be made about  $x$ ?
7. [Telescoping Series #14.44]
  - a) Compute  $\sum_{n=1}^N \frac{1}{n(n+1)}$  and then  $\sum_{n=1}^{\infty} \frac{1}{n(n+1)}$ .  
HINT: Use the *partial fraction decomposition*  $\frac{1}{(x-a)(x-b)} = \frac{1}{a-b} \left[ \frac{1}{x-a} - \frac{1}{x-b} \right]$ .
  - b) Use this to estimate  $\sum_{n=1}^{\infty} \frac{1}{n^2}$
8. [#14.45] Let  $S_n := a_1 + \cdots + a_n$ . If  $S_n = 1/n$  for all  $n \geq 1$ , find  $a_n$  for all  $n \geq 1$ .

[Last revised: October 26, 2013]