Homework Set 5 (Due in class on Thursday, Oct. 15)

(late papers accepted until 1:00 Friday)

Exam 1 will be held in class on Thursday, Oct. 22. You may use one 3×5 card with notes (on both sides). There will be no homework that week.

The problem numbers refer to the D'Angelo - West text.

1. A tennis ball is dropped from a height H. After each bounce it returns to two-thirds of its height on the previous bounce. How far does the ball travel until it is at rest on the floor?

2. [#14.2] For each condition below, give an example of an *unbounded* sequence such that $a_{n+1} - a_n > 0$ for all $n \in \mathbb{N}$ and the specified condition holds.
   a) $\lim (a_{n+1} - a_n) = 0$.
   b) $\lim (a_{n+1} - a_n)$ does not exist.
   c) $\lim (a_{n+1} - a_n) = L$, where $L > 0$.

3. Suppose that $x_0 = c$ for some real c and $x_{n+1} = \sqrt{1 + x_n^2}$ for all $n \in \mathbb{N}$. For which c does x_n converge?

4. [#14.9]. Proof or counterexample. Suppose that $x_n \to L$.
   a) For all $\epsilon > 0$, there exists $n \in \mathbb{N}$ such that $|x_{n+1} - x_n| < \epsilon$.
   b) There exists $n \in \mathbb{N}$ such that for all $\epsilon > 0$, $|x_{n+1} - x_n| < \epsilon$.
   c) There exists $\epsilon > 0$ such that for all $n \in \mathbb{N}$: $|x_{n+1} - x_n| < \epsilon$.
   d) For all $n \in \mathbb{N}$ there exists $\epsilon > 0$ such that $|x_{n+1} - x_n| < \epsilon$.

5. [#14.13] If $x_n \to L$, then every subsequence converges to L.

6. [#14.15] Let b and L be real numbers. If $b \le L + \epsilon$ for all $\epsilon > 0$, prove that $b \le L$.

7. If c is a complex number with $|c| < 1$, show that $(n^2+1)c^n \to 0$. Does $n^5 c^n$ converge?

8. [#14.18] If $a_1 = 1$ and $a_{n+1} = \sqrt{3a_n + 4}$ for $n \ge 1$, show that $a_n < 4$ for all $n \ge 1$.

9. [#14.32] A runaway train is hurtling toward a brick wall at a speed of 100 miles per hour. When it is 2 miles from the wall, a (speedy) fly begins to fly repeatedly between the train and the wall at the speed of 200 miles per hour. Determine how far the fly travels before it is smashed.

10. Show that $n^{\frac{1}{n}} \to 1$. [HINT: Let $x_n := n^{\frac{1}{n}} - 1$. Show that $x_n \to 0$ similarly to HW 4, Problem 7, by proving and using the inequality: if $a > 0$, then $(1+a)^n > 1 + na + \frac{n(n-1)}{2}a^2$].

[Last revised: October 15, 2009]