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Math 202 **Exam 2** Jerry L. Kazdan
December 10, 2009 9:00 — 10:20

DIRECTIONS: Part A has 4 shorter problems (5 points each) while Part B has 6 traditional problems (10 points each). To receive full credit your solution should be clear and correct. Neatness counts. You have 1 hour 20 minutes. Closed book, no calculators, but you may use one 3×5 card with notes on both sides.

PART A: Four shorter Problems, 5 points each.

A-1. Show that there is an $x > 0$ such that $x^5 = 17$.

Score	
A-1	
A-2	
A-3	
A-4	
B-1	
B-2	
B-3	
B-4	
B-5	
B-6	
Total	

A-2. Give an example of a bounded continuous function $f(x)$ for $x \in \mathbb{R}$ that does *not* attain its supremum.

A-3. Let f be a differentiable function with the property that $f'(x) = 0$ for all $x \in [0,3]$. Show that $f(x) =$ CONSTANT on the interval $[0,\ 3]$.

A-4. Let $f(x)$ be continuous with $\int_1^x f(t)\,dt = C + e^{(x-1)^2}$. Find f and the constant C.

PART B: Six traditional problems, 10 points each.

B-1. Let $f(x)$ be a differentiable function that in never zero. Use the definition of the derivative as the limit of a difference quotient to derive the usual formula for the derivative of $1/f(x)$.

B-2. Let a smooth function f have the properties:

$$f(0) = 4, \quad f(1) = 0, \quad f(3) = 6.$$

Show that at some point $0 < c < 3$ one has $f''(c) > 0$.

B-3. Use the definition of the integral as the limit of a Riemann sum to compute $\int_0^c x^2\,dx$ (here $c > 0$). [The formula $1^2 + 2^2 + \cdots + n^2 = \frac{1}{6}n(n+1)(2n+1)$ may be useful.]

B-4. Determine the disk of convergence of $\sum_{0}^{\infty} \frac{n^2(x-2)^{2n}}{4^n}$. [Only find the center and radius of this disk. You need not determine the convergence on the boundary of the disk.]

B-5. Suppose u is a twice differentiable function on $\mathbb{R}$ which satisfies the differential equation

$$\frac{d^2u}{dx^2} + b(x)\frac{du}{dx} - c(x)u = 0,$$

where $b(x)$ and $c(x)$ are continuous functions on $\mathbb{R}$ with $c(x) > 0$ for every $x \in [0,1]$.

a) Show that u cannot have a positive local maximum in the interval $(0,1)$. Also show that u cannot have a negative local minimum in $(0,1)$.

b) If $u(0) = u(1) = 0$, prove that $u(x) = 0$ for every $x \in [0,1]$.

B-6. Let I_k be closed bounded nested intervals, so $I_{k+1} \subseteq I_k$.

a) Use the completeness property of the real numbers ("bounded monotone sequences converge") to show that there is at least one point in the intersection, $\cap I_k$.

b) Give an example where the intersection is the interval $-1 \le x \le 1$.