For \( g(1) \) : (see §2 for details)
- ground field: \( k \)
- weight lattice: \( X = \mathbb{Z}^{k} \)
- group algebra: \( kX = k\langle x_1, x_2, \ldots, x_k \rangle \)
- Weyl group: \( W = S_k \)
- roots: \( \alpha_i, \beta_i \) for \( 1 \leq i \leq k \)
- positive roots: \( \alpha_i - \beta_i \) for \( 1 \leq i \leq k \)
- simple roots: \( \alpha_i = \epsilon_i - \epsilon_{i+1} \) for \( 1 \leq i \leq k-1 \)

**Def.** The Demazure-Lusztig operator is defined as:
\[
T_i = g(1) + (1-g(1)) \frac{e_i}{1-e_i} \quad 1 \leq i \leq k-1
\]

They generate an action of the tensor algebra \( H(1) \) on \( k[x_1, \ldots, x_k] \)

**Proof**
\[
T_i^2 = (1-g(1)) + \frac{e_i}{1-e_i} \left( T_i - e_i \right) = (1-g(1)) + \frac{e_i}{1-e_i} (T_i - e_i) = (1-g(1)) (1-g(1) - g(1) e_i) = (1-g(1)) = (1-g(1)) (1-g(1) - g(1) e_i)
\]

For \( w \in S_k \) with reduced expression \( w = s_{i_1} \cdots s_{i_l} \), define \( T_w = T_{i_l} \cdots T_{i_1} \) (well-defined because \( T_i \) satisfy the braid relations \( T_i T_j = T_j T_i \) for \( i \neq j \) and \( T_i^2 = T_i \) for \( i \neq j \)) from a left basis of the tensor algebra.

Set \( R_\alpha = R_\alpha(g(1)) \) (positive roots)
\[
Q = \mathbb{Z} \langle x_1, \ldots, x_k \rangle = \left\{ \sum_{i=1}^{k} a_i x_i : a_i \in \mathbb{Z}, \sum a_i = 0 \right\}, \quad \text{(root lattice)}
\]

\( Q = \mathbb{R} R_\alpha \) is the set of \( x \)-vectors (root vectors)

**Decol** \( (\lambda_1, \ldots, \lambda_k) \in \mathbb{Z} \): dominant if \( \lambda_1 \geq \lambda_2 \geq \cdots \geq \lambda_k \)
- regular if \( \lambda = \lambda \) \( \nu = \nu \)
- has maximum \( x \in \mathbb{Z} \)

For dominant weights, we define
\[
\lambda^\wedge = \max \{ \mu : (\lambda, \mu) \geq 0 \}
\]

(\( (\lambda, \mu) \) is an inner product, this is sometimes called dominance order on partitions)

\( (4,4,2,2) \geq (4,3,3,2) \) in dominance order

\( (4,3,2,2) - (4,3,2,2) = (0,0,0,0) = 0 \in Q \)

**Notation.** For \( \lambda \in \mathbb{Z}^k \), denote \( \lambda^\wedge = \text{dominant weight in the orbit } S_k \lambda \) (in formulas omitted if they are trivially dominant).

Let \( \text{conv}(S_k \lambda) \) be the convex hull of the orbit \( S_k \lambda \). In the case \( \lambda = \lambda \), the set of weights that occur with nonzero multiplicity in the irreducible character \( \chi_{\lambda} \),
e.g. $P_1^2 \lambda = \frac{f(1,0) - 1}{a + 1^2} \lambda \geq \frac{1}{a + 1^2}$ Take $\lambda = (1,0,0,0,0)$, $\lambda = \lambda_i$ (some result if we choose $\lambda = (2,1,0)$

Then $S_{\lambda} \lambda = \{ (1,0,0), (0,2,0) \}$. $\lambda = \lambda_i$.

$\lambda_i Q = \{ (1,0,0,0,0) \} \geq (a + 1^2)$

Hence orbit $S_{\lambda} \lambda$ has endpoints at $a = 0$ and $a = 1$

$\text{conv} (S_{\lambda} \lambda) = \{ (1,0,0,0,0), (0,2,0,0) \}$

Note that $\lambda_i Q = \{ (1,0,0,0,0) \} \geq (a + 1^2)$

Now that $\text{conv} (S_{\lambda} \lambda) = \text{conv} (S_{\lambda} \lambda_i)$, $\lambda_i \leq \lambda_i$.

$\lambda_i \leq \lambda_i$.

Note that $\lambda_i \leq \lambda_i$.

This conclusion is the transitive closure of the relation $S_{\lambda} \lambda \geq \lambda_i$ for $\lambda_i \lambda \geq \lambda_i$.

We extend this to all of $\mathbb{N}^5$ (instead of the wreath $S_{\lambda} \lambda$) by defining $\lambda \leq \lambda_i$ if $\lambda_i \lambda \geq \lambda_i$

e.g. $\lambda = (2,4,1,5,2)$, $\lambda_i = (1,3,1,2,0)$.

Hence $\lambda_i \lambda \geq \lambda_i$. 

Now $\lambda_i \lambda \geq \lambda_i$.

$\lambda_i \leq \lambda_i$.

Similarly, $(1,3,4,5,2) \geq (2,4,5,2) \geq (2,4,5,2)$.

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$\lambda_i \leq \lambda_i$.

Similarly, $(1,3,4,5,2) \geq (2,4,5,2) \geq (2,4,5,2)$.
Consider $\mu \leq \lambda$:

- If $\mu < \lambda$, since $(s, \mu) = (\emptyset, \mu)$, we have $(s, \mu) < (s, \lambda)$, and hence $s, \mu < \lambda$.

- If $\mu = (G, \lambda)$ and $\mu \leq \lambda$, then $\langle \omega, \mu \rangle > 0$ and hence $\mu \leq \lambda$.

If $(s, \mu) \leq \lambda$, then as $(s, \mu) > 0$, we know $(s, \mu) > 0$. Hence $(\omega, \mu) \geq 0$ and $\mu \leq \lambda$ with $\omega, \mu > 0$ means $s, \mu < \lambda$.

Hence $s, \mu < \lambda$.

As a result, the set $\{\kappa^\mu, \mu < \lambda\}$ is $\otimes$-bounded.

Given any not $v$, $(v, \lambda) \cap \{\mu, \mu < \lambda\}$ is compact if $v \leq \kappa$, $v, \kappa \leq \lambda$ for some $\kappa < \lambda$, $v \in \lambda$.

Hence $\kappa \{\kappa^\mu, \mu < \lambda\}$ is closed under $T_\lambda$.

For $\kappa < \lambda$, $\alpha$, and $\mu \leq \lambda$, we have:

$$T_\lambda (\kappa^\mu + \sum_{\alpha < \lambda} \kappa^\mu) = \kappa^\mu + \sum_{\alpha < \lambda} \kappa^\mu$$

for some $\mu \leq \lambda$. 