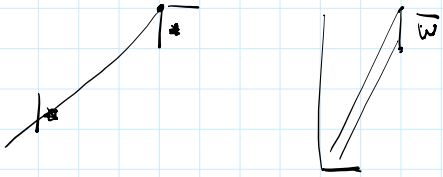
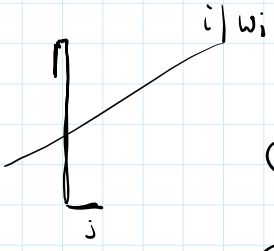
IPAC 2026 - Lecture 5

Thursday, 18 June 2026 18:00

$\# C_* + \# \tilde{C}_b - \# F_b^{\nwarrow} = \#$ intersections of some lines $= A$



inversions between some i,j st both $w_i < w_j$ are big and $i > j$



$(i,j) \rightsquigarrow A \iff v_j \le v_i < v_j + \frac{n}{m} + r'_{ij} + w_i - 1$ (a)

$(i,j) \rightsquigarrow F_b^{\nwarrow} \iff v_j + \frac{n}{m} + r'_{ij} + w_i - 1 - r''_{ij} \le v_i < v_j + \frac{n}{m} + k - 1$ (b)

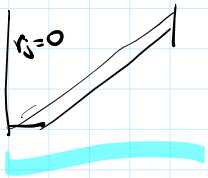
$(i,j) \rightsquigarrow C_* \iff v_j + \frac{n}{m} + r'_{ij} + w_i - 1 - r''_{ij} \le v_i < v_j + \frac{n}{m} + r'_{ij} + w_i - 1$ (c)

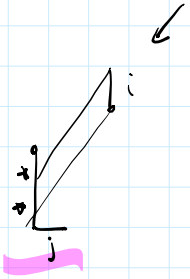
$(i,j) \in \tilde{C}_b \iff v_j \le v_i < v_j + \frac{n}{m} + k - 1$ (d)

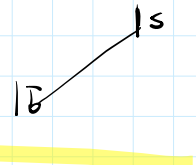
(a) $\vee$ (b) $\iff$ (d) (a) $\wedge$ (b) $\iff$ (c)

$\# C_+ + \# \tilde{C}_S = \# C_- + \# F_S^{\swarrow}$

$v_j \le v_i < v_j + \frac{n}{m} + r_j - 1$







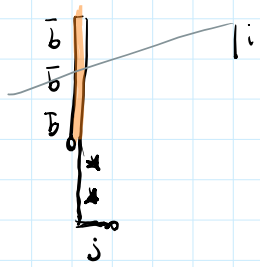
$v_i \le v_j$ $v_j + \frac{n}{m} < v_i + 1$ $v_j \le v_i$ $v_i + 1 < v_j + \frac{n}{m} + k$

by grouping steps with a big label with the following horiz. step

$$F_s' = \{ (i,j) \in V \times H \mid j<i,\ v_j + \tfrac{n}{m} + r_j - 1 \le v_i < v_j + \tfrac{n}{m} + k - 1 \}$$

$\updownarrow$

F_s''



$$\min\{0, \tfrac{n}{m} + r_j - 1\}$$

$$C_+ \cup \tilde{C}_s = C_- \cup F_s' = \{ (i,j) \in V \times H,\ j<i,\ v_j + r \le v_i < v_j + \tfrac{n}{m} + k - 1 \}$$

$v_i \le v_j \quad v_j + \frac{n}{m} + r_j < v_i + 1$ $\qquad$ $v_j \le v_i \quad v_i + 1 < v_j + \frac{n}{m} + k$

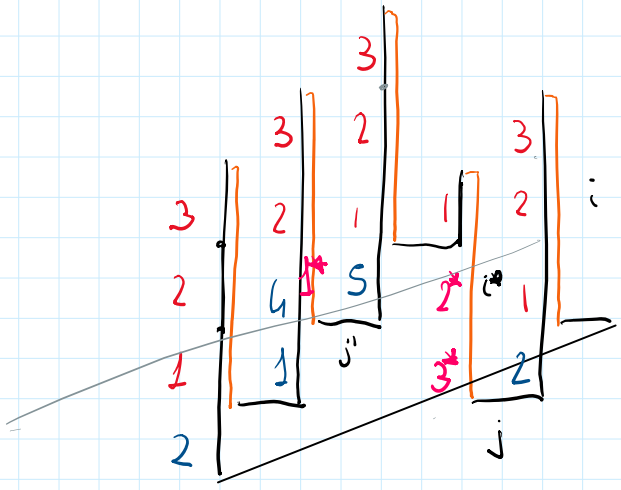
$\downarrow$

$v_i \le v_j < v_i + 1 - \frac{n}{m} - r_j$ $\qquad$ $v_j \le v_i < v_j + \frac{n}{m} + k - 1$

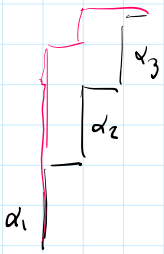
$$v_j - 1 + \tfrac{n}{m} + r_j < v_i \le v_j$$

$$\mathrm{dinv}(\tilde{\pi}) = \underbrace{\mathrm{tdinv}_{\mathrm{small}}(\tilde{\pi})}_{=} - \#\tilde{C}_s - \#\tilde{C}_b + \#\tilde{B}_s + \#\tilde{B}_b + \#F_s'' + \#F_b'' + \#F^{\nearrow}$$

$$\mathrm{dinv}(\pi) = \mathrm{tdinv}_{\mathrm{small}}(\pi) - \#C_- + \#C_+ + \#C_{\#} + \#B$$



$D_\alpha|_{q=1}^{\sim}$ path above an α-shape



cross stuff in: $H \cup D \cup S$ go from below to above

" : $V \cup B$ above to below

$$\frac{[n+k]_q}{[k]_q}\, \mathcal{O}_{c_k} \nabla w(p_n) \overset{?}{=} \sum_{\pi \in SQ(n+k)^{a \le k}} q^{\text{dinv}}\, t^{\text{area}}\, x^\pi$$

$$s_{\downarrow (n-1)^k}\, p_{n,n(k+1)} \overset{?}{=} \mathcal{O}_{c_k} \nabla w(p_n) \overset{?}{=} \sum_{\pi \in SQ(n+k)_{\text{dk}}} q^{\text{dinv}}\, t^{\text{area}}\, x^\pi$$

It would follow from the rational "rectangular" paths conj.