$\mathrm{dinv}(\tilde{\pi}) = \mathrm{dinv}(\pi)$

$\tilde{\pi}$ starting "tall" path

π ending decorated path

$$\mathrm{dinv}(\tilde{\pi}) = \mathrm{tdinv}_{small}(\tilde{\pi}) - \# \tilde{C}_s - \# \tilde{C}_b + \# \tilde{B}_s + \# \tilde{B}_b + \# F_s^{\swarrow} + \# F_b^{\swarrow} + \# F^{\nearrow}$$

$$\mathrm{dinv}(\pi) = \mathrm{tdinv}_{small}(\pi) - \# C_- + \# C_+ + \# C_* + \# B$$

- $\# C_+ - \# C_- = \# F_s^{\swarrow} - \# \tilde{C}_s$ (to show)
- $- \# \tilde{C}_b + \# \tilde{B}_b + \# F_b^{\swarrow} + \# F^{\nearrow} = \# C_*$

$\iff (\# C_* + \# \tilde{C}_b - \# F_b^{\swarrow}) - \# F^{\nearrow} = \# \tilde{B}_b$

$\# C_* + \# \tilde{C}_b - \# F_b^{\swarrow}$ counts the intersections between half-lines ℓ_i with steps in $\mathcal{H} \sqcup \mathcal{D} \sqcup \mathcal{S}$ belonging to π_{w_i} (the labels of those steps are less than the label of step i), summed over $i \in \mathcal{B}$

Idea: reduce this to counting pairs (i,j) $i \in \mathcal{B}$, $j \in \mathcal{H}$, $j < i$, satisfying certain constraints

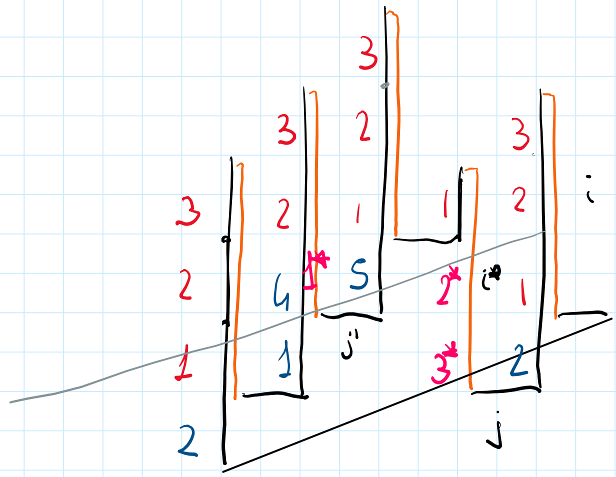
For (i,j) such pair, $r'_{ij} = \#$ decorated steps before j with label greater than w_i

There is exactly one decorated step labeled w_i. We call that i^* if that step shows up among these

$$v_{i^*} = v_j + \frac{n}{m} + r'_{ij} - 1$$

$V_{i^*} = V_j + \frac{n}{m} + r_{ij} - 1$

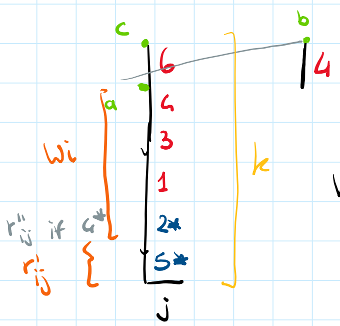
Let $r''_{ij} = \begin{cases} 1 & \text{if } i^* \text{ exists} \\ 0 & \text{otherwise} \end{cases}$



$V_j \le V_i < V_j + \frac{n}{m} + r'_{ij} + W_i - 1$ ⇐ condition to have an intersection

$\# F_b^{\swarrow} = \# \{ (i,j) \in \mathcal{B} \times \mathcal{B} \mid j < i,\ w_i < w_j,\ i \to j,\ V_i \le V_j < V_i + 1 \}$

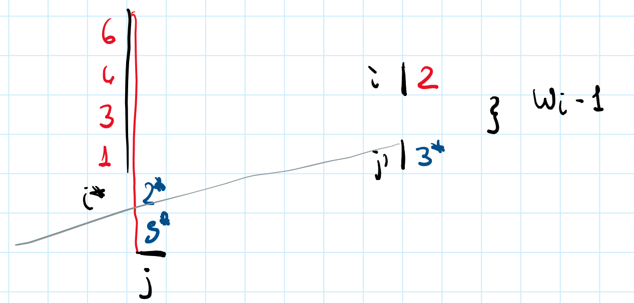
For $i \in \mathcal{B}$, $j \in \mathcal{H}$ $j < i$, $[\ \exists!\ i'$ st $(i, i') \in F_b^{\swarrow}$, with i' appearing above $j\ ]$ $\iff$



$a \le b < c$

$V_j + \frac{n}{m} + r'_{ij} + W_i - 1 + r''_{ij} \le V_i < V_j + \frac{n}{m} + k - 1$

Fix (i,j) st i^* is defined $(r''_{ij} = 1)$. We count contrib. to C^* from pairs (i^*, j'), $j' > i$ has at least one endpoint below i



$$v_i = v_{j'}^{+} + w_i - 1$$

$$v_{i^*} \leq v_{j'}^{+} < v_{i^*} + 1$$

$$v_{i^*} = v_j + \frac{n}{m} + r'_{ij}$$

$$v_j + \frac{n}{m} + r'_{ij} \leq v_i - w_i + 1 < v_j + \frac{n}{m} + r'_{ij} + 1$$

$$\Leftrightarrow \quad v_j + \frac{n}{m} + r'_{ij} + w_i - 1 \leq v_i < v_j + \frac{n}{m} + r'_{ij} + w_i$$

$$v_j + \frac{n}{m} + r'_{ij} + w_i - 1 - r''_{ij} \leq v_i < v_j + \frac{n}{m} + r'_{ij} + w_i - 1$$