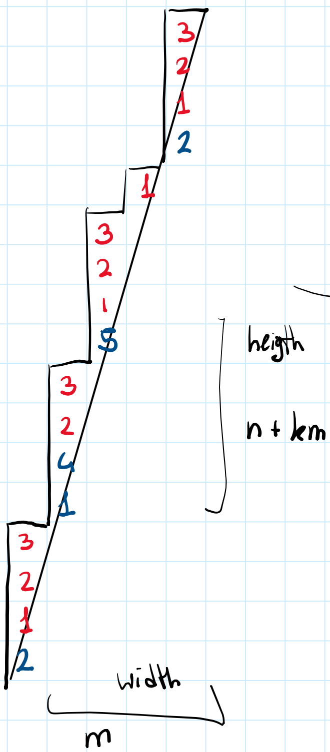
IPAC 2026 - Lecture 3

Thursday, 11 June 2026 18:00

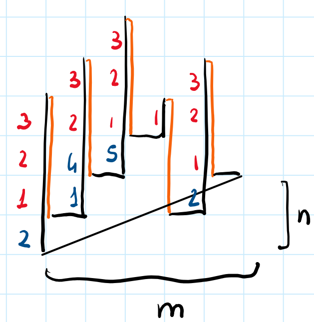


$m = 5$

$n = 2$

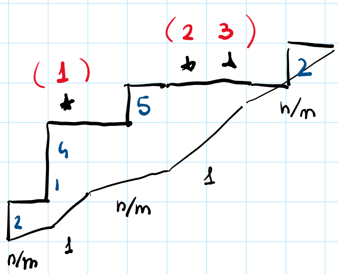
$k = 3$

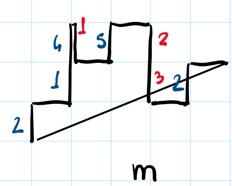
EJS repr.



$\tilde{\pi}$

$1 < 2 < \cdots < 1 < 2 < \dots$

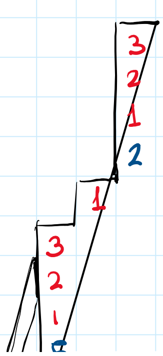




π

n

dinv ?



dinv here ?

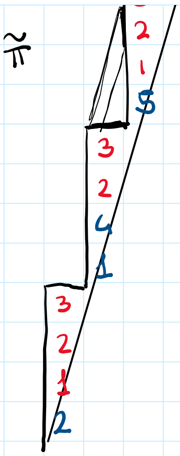
Rmk: if $k \geq 1$, the path is taller than it's wide

$\tilde{\pi}$

$a_i \in \mathbb{Q}$

[illegible]

$\tilde{\pi}$



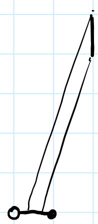
$a_i \in \mathbb{Q}$

tdinv : $\# \{ (i,j) \mid (a_i, i) \le_{lex} (a_j, j) \le_{lex} (a_i + k + \frac{n}{m}, i) \text{ and } w_i < w_j \}$

$= \# \{ (i,j) \mid (v_i, i) \le_{lex} (v_j, j) \le_{lex} (v_i + 1, i) \text{ and } w_i < w_j \}$

" $i \to j$ "

$\frac{n+km}{m} = k + \frac{n}{m}$



cdinv $= - \# \{$ occurrences of this pattern $\}$

$+ \# \{ i \mid a_i < 0 \}$

$\mathrm{dinv}(\tilde{\pi}) = \mathrm{tdinv}(\tilde{\pi}) + \mathrm{cdinv}(\tilde{\pi})$

$\mathcal{V}$ = { set of vertical "small-labeled" steps of $\tilde{\pi}$ }

$\mathcal{H}$ = { set " horizontal steps of π }

$\mathcal{B}$ = { set of vertical "big-labeled" steps of $\tilde{\pi}$ } ×

$\mathcal{D}$ = { " decorated south steps of $\tilde{\pi}$ }

$\mathcal{S}$ = { non-dec south steps of $\tilde{\pi}$ } ×

π

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if comes before i in the path

$\tilde{C}_s = \{ (i,j) \in \mathcal{V} \times \mathcal{H} \mid j < i \text{ and } v_j \le v_i < v_j + k + \frac{n}{m} - 1 \}$

$\tilde{C}_b = \{ (i,j) \in \mathcal{B} \times \mathcal{H} \mid j < i \text{ and } v_j \le v_i < v_j + k + \frac{n}{m} - 1 \}$

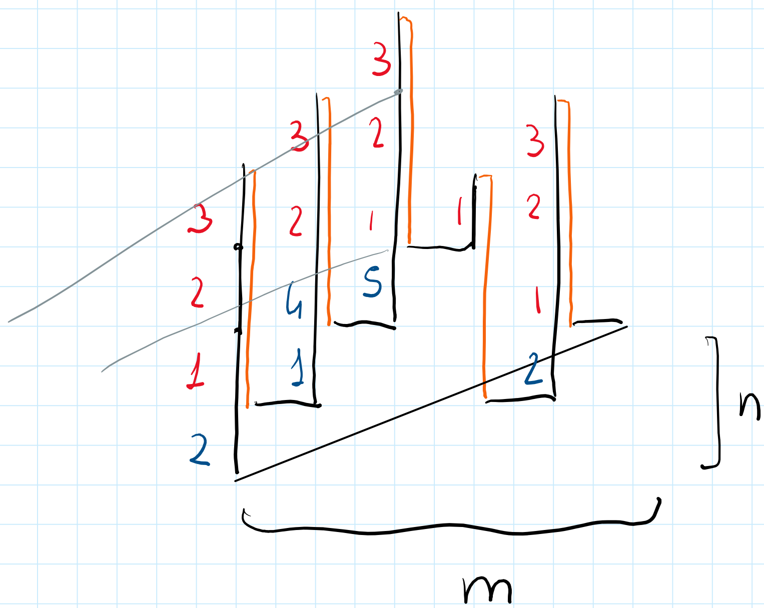
$\tilde{B}_s = \{ i \in \mathcal{V} \mid v_i < 0 \}$

$\tilde{B}_b = \{ i \in \mathcal{B} \mid v_i < 0 \}$

$$F_s^{\swarrow} = \{ (i,j) \in V \times B \mid j < i \text{ and } i \to j \}$$

$$F_b^{\swarrow} = \{ (i,j) \in B \times B \mid j < i,\ w_i < w_j,\ i \to j \}$$

$$F^{\nearrow} = \{ (i,j) \in (V \cup B) \times B \mid i < j,\ w_i < w_j,\ i \to j \}$$



$\pi_{\bar{a}}$ is the path obtained from $\tilde{\pi}$ by removing all big steps with label $> \bar{a}$, so $\pi_{\bar{k}} = \tilde{\pi}$, $\pi_{\bar{0}} = \pi$

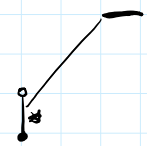
- $i \in B$, then $i \in \tilde{B}_b \iff \ell_i$ starts below the path

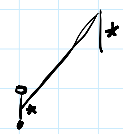
- $(i,j) \in (V \cup B) \times B$, $(i,j) \in F^{\nearrow} \iff i \in \pi_{w_j}$ and ℓ_j intersects i (above $\iff$: $w_i < w_j$)

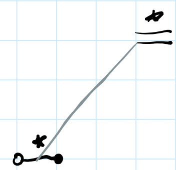
- The number of intersections of half-lines ℓ_i with steps in $H \cup D \cup S$ belonging to $\pi_{\bar{i}}$, summed over $i \in B$, is counted by belonging to π_i, summed over $i \in B$, is counted by

$$\# \tilde{C}_b - \# F_b^{\swarrow} + \# C^*$$

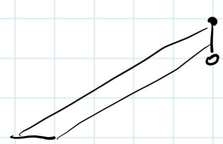
$C^* = \#$ patterns like







• $\# C_+ - \# C_- = \# F_s^{\swarrow} - \# \tilde{C}_s$



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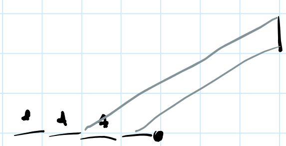

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