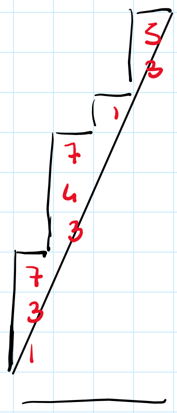
IPAC 2026 - Lecture 2

Thursday, 4 June 2026 18:00

$$s_{(m-1)^k}^{\perp}\, e_{m,n+mk} \;\leftarrow\; \frac{[m]_q}{[d]_q}\, p_{m,n+mk} \qquad (\text{True if } (m,n)=1, \text{ conjectural otherwise})$$



$n=5$

$k=1$

$m=4$

Recall: $$e_{m,n+mk} = \sum_{\pi\in LD(m,n+mk)} q^{dinv(\pi)}\, t^{area(\pi)}\, x^{\pi}$$

$$e_{m,n} = F_{m,n}(e_d) \qquad p_{m,n} = F_{m,n}(p_d)$$

$(m,n) = d$

$f, g \in \Lambda$

$$g^{\perp} f = g[Y]^{\perp} f[X+Y]\Big|_{Y=0}$$

$$g = h_\alpha \qquad \langle h_\alpha, m_\beta \rangle = \delta_{\alpha\beta}$$

GGG '24 : $$s_{(m-1)^k} = \sum_{\alpha \text{ allowable}} (-1)^{sgn(\alpha)}\, h_{\bar{\alpha}} \quad (\text{mod } h_j \mid j > m)$$

$\bar{\alpha}_i = m - \alpha_i \qquad sgn(\alpha) = \#\{1 \le i \le k \mid \alpha_i = 0\} \qquad \alpha \vDash k \qquad \ell(\alpha) = k$

$\alpha \vDash k$ is allowable if it is the content of a possible placement of k competitors in a tournament with ties allowed.

allowable

$k=6$ 1 2 3 4 5 6 ↓

2 1 2 6 5 2 $\leftarrow$ Fubini ranking $\alpha = (1, 3, 0, 0, 1, 1)$

2 1 2 6 5 2	$p = 6$	1	$r = 2$	2 1 2 5 5 2
3 1 1 4 6 4	$p = 3$	2	$r = 4$	3 1 1 5 6 4
1 3 5 6 1 3	$\nexists p$	2	$r = 3$	1 4 5 6 1 3
3 1 4 6 5 2	$p = 6$	1	$\nexists r$	3 1 4 5 5 2

Sign-reversing involution:

- Let p be the largest entry in the word s.t. there is only one p, and if q is the largest entry less than p, then p is to the left of every q
- Let r be the largest repeated entry in the word

1) $p > r$ or $\nexists r$: replace p with q
2) $p < r$ or $\nexists p$: replace the first r with $s-1$, where s is the smallest entry in the word larger than r (or $s = k+1$ if r is max)
3) $\nexists p$ $\nexists r$ $\Rightarrow$ $a = 1\,2\,3\,4\,5\,6$: do nothing

φ is a sign-reversing involution, with $1 \cdots k$ as only fixed point

it also preserves tied inversions: $i < j \Rightarrow (a_i \geq a_j \Leftrightarrow \varphi(a)_i \geq \varphi(a)_j)$

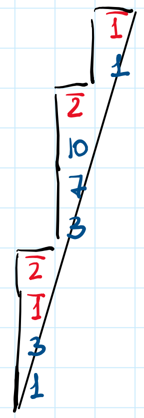
$$S_{(m-1)^k} = \sum_{\alpha \text{ allowable}} (-1)^{\mathrm{sgn}(\alpha)} h_{\bar{\alpha}}$$

$\alpha \vDash 2$ $\alpha = (1,1)$

$\tilde{\alpha} \vDash 4$ $\tilde{\alpha} = (2,2)$

$n = 4$

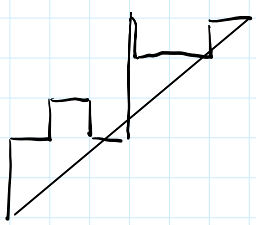
$k = 2$



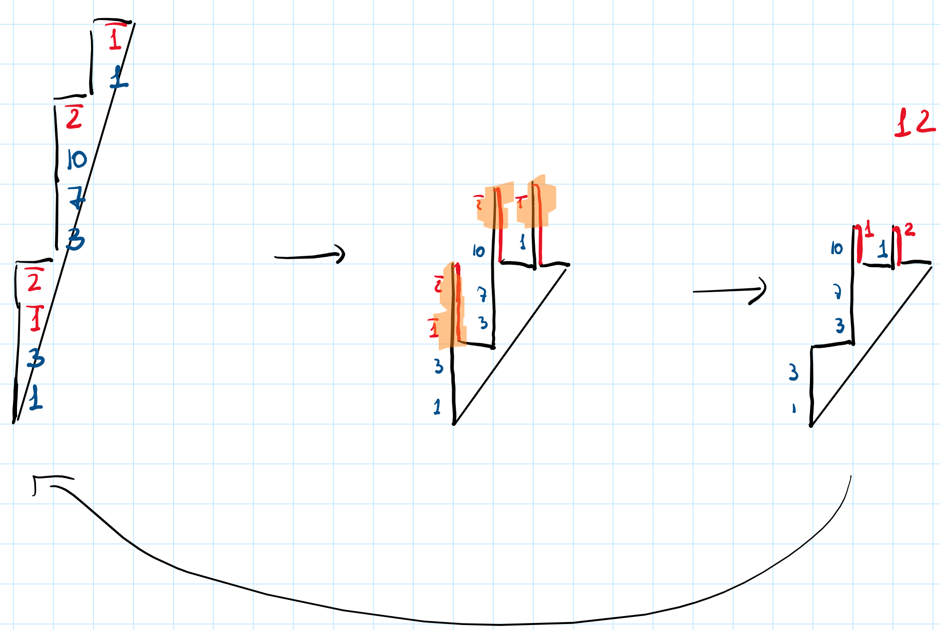
$\underbrace{1 < 2 < \dots}_{X} , < \underbrace{\bar{1} < \bar{2} < \bar{3}}_{Y}$

$m = 3$

ENS-repr. East, North, South exactly k times, V SN



$s^{\perp}_{(m-1)^k} e_{m, n+mk}$



This map preserves the vertical distances between the endpoints of steps and the main diagonal.

$$(a_i, i) \leq_{lex} (a_j, j) \leq_{lex} (a_i + 1, i)$$

$$\Updownarrow$$

verti distance

$$(v_i, i) \leq_{lex} (V_j, j) \leq_{lex} (v_i + \frac{n}{m}, i)$$